BÀI 6 +7: BIẾN ĐỔI ĐƠN GIẢN BIỂU THỨC CHỨA CĂN BẬC HAI

I. TÓM TẮT LÝ THUYẾT

1. Đưa thừa số ra ngoài dấu căn

$$\sqrt{A^2B}=|A|\sqrt{B}(B\ge 0)=\begin{cases}A\sqrt{B}(A\ge 0;B\ge 0)\\-A\sqrt{B}(A<0;B\ge 0)\end{cases}$$

2. Đưa thừa số vào trong dấu căn

$$A\sqrt{B}=\begin{cases}\sqrt{A^2B}(khiA\ge 0;B\ge 0)\\-\sqrt{A^2B}(khiA<0;B\ge 0)\end{cases}$$

3. Khử mẫu của biểu thức lấy căn

$$\sqrt{\frac{A}{B}}=\sqrt{\frac{AB}{B^2}}=\frac{1}{|B|}\sqrt{AB}(B\ne 0;AB\ge 0)$$

4. Trục căn thức ở mẫu

+) $\frac{A}{\sqrt{B}}=\frac{A\sqrt{B}}{B}(B>0)$ +) $\frac{m}{\sqrt{A}\pm\sqrt{B}}=\frac{m(\sqrt{A}\mp\sqrt{B})}{A-B}(A\ge 0;B\ge 0;A\ne B)$

+) $\frac{m}{\sqrt{A}\pm B}=\frac{m(\sqrt{A}\mp B)}{A-B^2}(A\ge 0;A\ne B^2)$

II. CÁC DẠNG BÀI TẬP

Dạng 1: Đưa thừa số ra ngoài dấu căn hoặc vào trong dấu căn

Phương pháp giải: Sử dụng kiến thức sau

Cách đưa thừa số A^2 ra ngoài dấu căn: $\sqrt{A^2B}=|A|\sqrt{B}(B\ge 0)=\begin{cases}A\sqrt{B}(A\ge 0;B\ge 0)\\-A\sqrt{B}(A<0;B\ge 0)\end{cases}$

Cách đưa thừa số vào trong dấu căn: $A\sqrt{B}=\begin{cases}\sqrt{A^2B}(khiA\ge 0;B\ge 0)\\-\sqrt{A^2B}(khiA<0;B\ge 0)\end{cases}$

Bài 1: Viết gọn các biểu thức sau

a. $\sqrt{25.90}$ b. $\sqrt{96.125}$ c. $\sqrt{75.54}$ d. $\sqrt{245.35}$

HD:

a. $\sqrt{25.90}=15\sqrt{10}$ b. $\sqrt{96.125}=\sqrt{16.6.5.25}=20\sqrt{30}$

c. $\sqrt{75.54}=45\sqrt{2}$ d. $\sqrt{245.35}=\sqrt{49.5.5.7}=35\sqrt{7}$

Bài 2: Đưa nhân tử ra ngoài dấu căn:

a) $\sqrt{54}$ b) $\sqrt{108}$ c) $0,1\sqrt{20000}$ d) $-0,05\sqrt{28800}$

HD:

a) $\sqrt{54}=\sqrt{9.6}=3\sqrt{6}$

b) $\sqrt{108}=\sqrt{36.3}=6\sqrt{3}$

c) $0,1\sqrt{20000}=0,1\sqrt{2.10000}=0,1.100\sqrt{2}=10\sqrt{2}$

d) $-0,05\sqrt{28800}=-0,05\sqrt{144.100.2}=-0,05.12.10\sqrt{2}=-6\sqrt{2}$

Bài 3:Đưa thừa số ra ngoài dấu căn

a. $\sqrt{27x^2}\,(x\ge 0)$ b. $\sqrt{8xy^2}\,(x\ge 0;y\le 0)$

c. $\sqrt{25x^3}\,(x>0)$ d. $\sqrt{48xy^4}\,(x\ge 0;y\in R)$

e. $\sqrt{48y^4}$ f) $\sqrt{8y^2}$ với y > 0

HD:

a) $\sqrt{27x^2}=\sqrt{(3x)^2.3}=|3x|\sqrt{3}=3\sqrt{3}x\,(x\ge 0)$

b) $\sqrt{8xy^2}=\sqrt{(2y)^2.2x}=|2y|\sqrt{2x}=-2y\sqrt{2x}\,(x\ge 0;y\le 0)$

c) $\sqrt{25x^3}=5x\sqrt{x}\,(x>0)$ d. $\sqrt{48xy^4}\,(x\ge 0;y\in R)=4y^2\sqrt{3x}\,(y^2\ge 0)$

d) $\sqrt{48xy^4}=|y^2|\sqrt{16.3x}=4y^2\sqrt{3x}$

e) $\sqrt{48y^4}=\sqrt{16.3.(y^2)^2}=4y^2\sqrt{3}$

f) $\sqrt{8y^2} = 2|y|\sqrt{2} = 2y\sqrt{2}$ (vì y>0)

Bài 4: Đưa nhân tử vào trong dấu căn:

a) $3\sqrt{5}$ b) $-5\sqrt{2}$ c) $2\sqrt{2}$ d) $3\sqrt{2}$

HD:

a) $3\sqrt{5} = \sqrt{3^2.5} = \sqrt{45}$ b) $-5\sqrt{2} = -\sqrt{5^2.2} = -\sqrt{50}$

c) $2\sqrt{2} = \sqrt{2^2.2} = \sqrt{8}$ d) $3\sqrt{2} = \sqrt{3^2.2} = \sqrt{18}$

Bài 5: Đưa nhân tử vào trong dấu căn:

a) $-\frac{2}{3}\sqrt{xy}$ b) $x\sqrt{5}$ với $x \ge 0$ c) $x\sqrt{13}$ với $x < 0$ d) $x\sqrt{\frac{2}{x}}$ với $x > 0$

e) $a\sqrt{13}\,(a \ge 0)$ f) $a\sqrt{\frac{-15}{a}}\,(a < 0)$ g) $\frac{a}{2}\sqrt{\frac{12}{a}}\,(a > 0)$ h) $a\sqrt{2}\,(a \le 0)$

HD:

a) $-\frac{2}{3}\sqrt{xy} = -\sqrt{\left(\frac{2}{3}\right)^2 xy} = -\sqrt{\frac{4}{9}xy}$ b) $x\sqrt{5} = \sqrt{5x^2}$ (vì $x \ge 0$)

c) $x\sqrt{13} = -\sqrt{13x^2}$ vì $x < 0$ d) $x\sqrt{\frac{2}{x}} = \sqrt{\frac{2x^2}{x}} = \sqrt{2x}$ vì $x > 0$

e) $a\sqrt{13} = \sqrt{13a^2}\,(a \ge 0)$

f) $a\sqrt{\frac{-15}{a}} = -(-a)\sqrt{\frac{-15}{a}} = -\sqrt{\frac{-15a^2}{a}} = -\sqrt{-15a}\,(a < 0)$

g) $\frac{a}{2}\sqrt{\frac{12}{a}} = \sqrt{3a}\,(a > 0)$ h) $a\sqrt{2} = -\sqrt{2a^2}\,(a \le 0)$

Dạng 2: So sánh các căn bậc hai

Phương pháp giải: Đưa thừa số ra ngoài hoặc vào trong dấu căn rồi so sánh

Bài 1: So sánh hai số sau (không dùng máy tính):

a) $3\sqrt{3}$ và $\sqrt{12}$ b) 20 và $3\sqrt{5}$

c) $\frac{1}{3}\sqrt{54}$ và $\frac{1}{5}\sqrt{150}$ d) $\frac{1}{2}\sqrt{6}$ và $6\sqrt{\frac{1}{2}}$

e) $\frac{\sqrt{5}}{3\sqrt{7}+5\sqrt{2}}$ và $\frac{3}{13}$

f) $\sqrt{30}-\sqrt{29}$ và $\sqrt{29}-\sqrt{28}$

g) $\sqrt{2012}+\sqrt{2014}$ và $2\sqrt{2013}$

h) $\sqrt{2014}-\sqrt{2013}$ và $\sqrt{2013}-\sqrt{2012}$

HD:

a) Ta có: $3\sqrt{3}=\sqrt{27};\sqrt{12}=\sqrt{12}$

Vì $27>12$ nên $\sqrt{27}>\sqrt{12}$. Vậy $3\sqrt{3}>\sqrt{12}$.

b) Ta có: $\sqrt{20}=2\sqrt{5}$

Vì $2<3$ nên $2\sqrt{5}<3\sqrt{5}$. Vậy $\sqrt{20}<3\sqrt{5}$.

c) Ta có: $\frac{1}{3}\sqrt{54}=\sqrt{6}$; $\frac{1}{5}\sqrt{150}=\sqrt{6}\Rightarrow\frac{1}{3}\sqrt{54}=\frac{1}{5}\sqrt{150}$

d) Ta có: $\frac{1}{2}\sqrt{6}=\sqrt{\frac{3}{2}}$; $6\sqrt{\frac{1}{2}}=\sqrt{18}$

Vì $\frac{3}{2}<18$ nên $\sqrt{\frac{3}{2}}<\sqrt{18}$. Vậy $\frac{1}{2}\sqrt{6}<6\sqrt{\frac{1}{2}}$.

e) Ta có: $\frac{\sqrt{5}}{3\sqrt{7}+5\sqrt{2}}=\frac{\sqrt{5}.\left(3\sqrt{7}-5\sqrt{2}\right)}{13}=\frac{3\sqrt{35}-5\sqrt{10}}{13}$;

Ta thấy $3\sqrt{35}<3\sqrt{36};5\sqrt{10}<5\sqrt{9}\Rightarrow 3\sqrt{35}-5\sqrt{10}<3$

Vậy $\frac{\sqrt{5}}{3\sqrt{7}+5\sqrt{2}}<\frac{3}{13}$

f) Ta có: $\sqrt{30}-\sqrt{29}=\frac{1}{\sqrt{30}+\sqrt{29}}$; $\sqrt{29}-\sqrt{28}=\frac{1}{\sqrt{29}+\sqrt{28}}$

Vì $\sqrt{30}+\sqrt{29}>\sqrt{29}+\sqrt{28}\Rightarrow\frac{1}{\sqrt{30}+\sqrt{29}}<\frac{1}{\sqrt{29}+\sqrt{28}}$

Vậy $\sqrt{30}-\sqrt{29}<\sqrt{29}-\sqrt{28}$.

g) Ta có: $\left(\sqrt{2012}+\sqrt{2014}\right)^2=4026+2\sqrt{2012.2014}=4026+2\sqrt{2013^2-1}$

$\left(2\sqrt{2013}\right)^2=8052=4026+2\sqrt{2013^2}$

Vì $2013^2-1<2013^2$ nên $\left(\sqrt{2012}+\sqrt{2014}\right)^2<\left(2\sqrt{2013}\right)^2$

Vậy $\sqrt{2012}+\sqrt{2014}<2\sqrt{2013}$.

h) Ta có: $\sqrt{2014}-\sqrt{2013}=\frac{1}{\sqrt{2014}+\sqrt{2013}}$; $\sqrt{2013}-\sqrt{2012}=\frac{1}{\sqrt{2013}+\sqrt{2012}}$

Vì $\sqrt{2014}+\sqrt{2013}>\sqrt{2013}+\sqrt{2012}$ nên $\sqrt{2014}-\sqrt{2013}<\sqrt{2013}-\sqrt{2012}$

Bài 2: So sánh hai số sau (không dùng máy tính):

a) $\sqrt{2}+\sqrt{3}$ và $\sqrt{10}$

b) $\sqrt{3}+2$ và $\sqrt{2}+\sqrt{6}$

c) 16 và $\sqrt{15}.\sqrt{17}$

d) 8 và $\sqrt{15}+\sqrt{17}$

e) $2\sqrt{29}$ và $4\sqrt{3}$

f) $\frac{5}{4}\sqrt{2}$ và $\frac{3}{2}\sqrt{\frac{3}{2}}$

g) $5\sqrt{2}$ và $3\sqrt{13}$

h) $\frac{5}{2}\sqrt{\frac{1}{6}}$ và $6\sqrt{\frac{1}{37}}$

HD:

a) $\sqrt{2}+\sqrt{3}$ và $\sqrt{10}$.

Ta có: $\left(\sqrt{2}+\sqrt{3}\right)^2=5+2\sqrt{6}<5+2\sqrt{\frac{25}{4}}=5+2.\frac{5}{2}=10$.

Do đó $\sqrt{2}+\sqrt{3}<10$.

b) $\sqrt{3}+2$ và $\sqrt{2}+\sqrt{6}$.

Ta có : $\left(\sqrt{3}+2\right)^2=7+4\sqrt{3}$.

$\left(\sqrt{2}+\sqrt{6}\right)^2=8+2\sqrt{12}=8+4\sqrt{3}>7+4\sqrt{3}$.

Do đó $\sqrt{2}+\sqrt{6}>\sqrt{3}+2$.

c) 16 và $\sqrt{15}.\sqrt{17}$

Ta có: $\sqrt{15}.\sqrt{17}=\sqrt{15.17}=\sqrt{(16-1)(16+1)}=\sqrt{16^2-1}<\sqrt{16^2}=16$.

Do đó $\sqrt{15}.\sqrt{17}<16$.

d) 8 và $\sqrt{15}+\sqrt{17}$.

Ta có: $\left(\sqrt{15}+\sqrt{17}\right)^2=32+2\sqrt{15.17}$.

$8^2 = 32 + 2.16 > 32 + 2.\sqrt{15.17}$.

Do đó $8 > \sqrt{15} + \sqrt{17}$.

e) Ta có $\begin{cases} 2\sqrt{29} = \sqrt{2^2.29} = \sqrt{116} \\ 3\sqrt{13} = \sqrt{117} \end{cases} \Rightarrow 2\sqrt{29} < 3\sqrt{13}$

f) $\begin{cases} \frac{5}{4}\sqrt{2} = \sqrt{\left(\frac{5}{4}\right)^2.2} = \sqrt{\frac{25}{8}} \\ \frac{3}{2}\sqrt{\frac{3}{2}} = \sqrt{\left(\frac{3}{2}\right)^2.\frac{3}{2}} = \sqrt{\frac{27}{8}} \end{cases} \Rightarrow \frac{5}{4}\sqrt{2} < \frac{3}{2}\sqrt{\frac{3}{2}}$

g) Ta có: $\left.\begin{matrix} 5\sqrt{2} = \sqrt{50} \\ 4\sqrt{3} = \sqrt{48} \end{matrix}\right\} \rightarrow 4\sqrt{3} < 5\sqrt{2}$

h) Ta có: $\left.\begin{matrix} \frac{5}{2}\sqrt{\frac{1}{6}} = \sqrt{\frac{25}{24}} \\ 6\sqrt{\frac{1}{37}} = \sqrt{\frac{36}{37}} \end{matrix}\right\} \rightarrow \frac{5}{2}\sqrt{\frac{1}{6}} > 6\sqrt{\frac{1}{37}} \left(\frac{25}{4} > 1 > \frac{36}{37}\right)$

Bài 3:Sắp xếp các số sau theo thứ tự tăng dần:

a) $3\sqrt{5}; 2\sqrt{6}; \sqrt{29}; 4\sqrt{2}$ b) $2\sqrt{5}$, $2\sqrt{6}$, $\sqrt{29}$, $3\sqrt{5}$ c) $3\sqrt{6}$, $3\sqrt{3}$, $4\sqrt{7}$, $2\sqrt{14}$

HD:

a) Ta có: $3\sqrt{5} = \sqrt{45}; 2\sqrt{6} = \sqrt{24}; 4\sqrt{2} = \sqrt{32} \rightarrow 2\sqrt{6} < \sqrt{29} < 4\sqrt{2} < 3\sqrt{5}$

b) Ta có: $2\sqrt{5} = \sqrt{20}; 2\sqrt{6} = \sqrt{24}; \sqrt{29}; 3\sqrt{5} = \sqrt{45}$

Sắp xếp $2\sqrt{5}; 2\sqrt{6}; \sqrt{29}; 3\sqrt{5}$

c) Ta có: $3\sqrt{6} = \sqrt{54}; 3\sqrt{3} = \sqrt{27}; 4\sqrt{7} = \sqrt{112}; 2\sqrt{14} = \sqrt{56}$

Sắp xếp $3\sqrt{3}; 3\sqrt{6}; 2\sqrt{14}; 4\sqrt{7}$.

Bài 4:Sắp xếp các số sau theo thứ tự giảm dần: $7\sqrt{2}; 2\sqrt{8}; \sqrt{28}; 5\sqrt{2}$

HD:

Ta có: $2\sqrt{8} = 4\sqrt{2}; \sqrt{28} = 2\sqrt{7} \Rightarrow 7\sqrt{2} > 5\sqrt{2} > 2\sqrt{8} > \sqrt{28}$

Dạng 3: Rút gọn biểu thức chứa căn bậc hai

Phương pháp giải: Đưa thừa số ra ngoài hoặc vào trong dấu căn rồi rút gọn

Bài 1: Thực hiện tính:

a) $A=5\sqrt{8}+\sqrt{50}-2\sqrt{18}$

b) $H=\sqrt{(3+\sqrt{5})^2}+\sqrt{5}$

c) $A=\sqrt{27}-2\sqrt{12}-\sqrt{75}$

d) $A=2\sqrt{3}-4\sqrt{27}+5\sqrt{48}$

e) $P=\sqrt{8}-\sqrt{18}+2\sqrt{32}$

f) $M=(3\sqrt{50}-5\sqrt{18}+3\sqrt{8})\sqrt{2}$

HD:

a) $A=5\sqrt{4.2}+\sqrt{25.2}-2\sqrt{9.2}=5.2\sqrt{2}+5\sqrt{2}-2.3\sqrt{2}=10\sqrt{2}+5\sqrt{2}-6\sqrt{2}$
$=(10+5-6)\sqrt{2}=9\sqrt{2}$

b) $H=\sqrt{(3-\sqrt{5})^2}+\sqrt{5}=|3-\sqrt{5}|+\sqrt{5}=3-\sqrt{5}+\sqrt{5}=3$

c) $A=\sqrt{9.3}-2\sqrt{9.3}-\sqrt{25.3}=\sqrt{3^2.3}-2\sqrt{3^2.3}-\sqrt{5^2.3}=3\sqrt{3}-4\sqrt{3}-5\sqrt{3}=-6\sqrt{3}$

d) $A=2\sqrt{3}-4\sqrt{9.3}+5\sqrt{16.3}=2\sqrt{3}-4\sqrt{3^2.3}+5\sqrt{4^2.3}=2\sqrt{3}-12\sqrt{3}+20\sqrt{3}=10\sqrt{3}$

e) $P=\sqrt{2^2.2}-\sqrt{3^2.2}+2\sqrt{4^2.2}=2\sqrt{2}-3\sqrt{2}+8\sqrt{2}=7\sqrt{2}$

f) $M=(3\sqrt{25.2}-5\sqrt{9.2}+3\sqrt{4.2})\sqrt{2}$
$=(3\sqrt{5^2.2}-5\sqrt{3^2.2}+3\sqrt{2^2.2})\sqrt{2}$
$=(15\sqrt{2}-15\sqrt{2}+6\sqrt{2})\sqrt{2}$
$=6\sqrt{2}.\sqrt{2}=12$

Bài 2: Thực hiện phép tính:

a) $A=\sqrt{125}-4\sqrt{45}+3\sqrt{20}-\sqrt{80}$

b) $B=\sqrt{20}-\sqrt{45}+2\sqrt{5}$

c) $A=\sqrt{3}(\sqrt{27}+4\sqrt{3})$

d) $P=\sqrt{2}(\sqrt{8}-2\sqrt{3})+2\sqrt{6}$

e) $3\sqrt{20}+\sqrt{45}-2\sqrt{80}$

f) $2\sqrt{32}-5\sqrt{27}-4\sqrt{8}+3\sqrt{75}$

g) $B=2\sqrt{3}+3\sqrt{27}-\sqrt{300}$

h) $A=3\sqrt{2}+4\sqrt{18}$

HD:

a) $A=\sqrt{25.5}-4\sqrt{9.5}+3\sqrt{4.5}-\sqrt{16.5}=\sqrt{5^2.5}-4\sqrt{3^2.5}+3\sqrt{2^2.5}-\sqrt{4^2.5}$
$=5\sqrt{5}-12\sqrt{5}+6\sqrt{5}-4\sqrt{5}=-5\sqrt{5}$

b) $B=\sqrt{2^2.5}-\sqrt{3^2.5}+2\sqrt{5}=2\sqrt{5}-3\sqrt{5}+2\sqrt{5}=\sqrt{5}$

c) $A=\sqrt{3}(\sqrt{27}+4\sqrt{3})=\sqrt{81}+4\sqrt{9}=\sqrt{9^2}+4\sqrt{3^2}=9+4.3$

d) $P=\sqrt{2}(\sqrt{8}-2\sqrt{3})+2\sqrt{6}=\sqrt{16}-2\sqrt{6}+2\sqrt{6}=\sqrt{4^2}-2\sqrt{6}+2\sqrt{6}=4$

e) $3\sqrt{20}+\sqrt{45}-2\sqrt{80}=3\sqrt{4.5}+\sqrt{9.5}-2\sqrt{16.5}=3\sqrt{2^2.5}+\sqrt{3^2.5}-2\sqrt{4^2.5}=6\sqrt{5}+3\sqrt{5}-8\sqrt{5}$
$=\sqrt{5}$

f) $=2\sqrt{4^2.2}-5.\sqrt{3^2.3}-4.\sqrt{2^2.2}+3.\sqrt{5^2.3}= =8\sqrt{2}-15\sqrt{3}-8\sqrt{2}+15\sqrt{3}$ = 0

g) $B=2\sqrt{3}+3\sqrt{9.3}-\sqrt{100.3}$
$=2\sqrt{3}+3\sqrt{3^2.3}-\sqrt{10^2.3}$
$=2\sqrt{3}+3.3.\sqrt{3}-10\sqrt{3}$
$=\sqrt{3}$

h) $A=3\sqrt{2}+4\sqrt{9.2}=3\sqrt{2}+4\sqrt{3^2.2}$
$A=3\sqrt{2}+12\sqrt{2}$
$A=15\sqrt{2}$

Bài 3: Rút gọn các biểu thức sau:

a) $5\sqrt{\frac{1}{5}}+\frac{1}{2}\sqrt{20}+\sqrt{5}$

b) $\sqrt{\frac{1}{2}}+\sqrt{4,5}+\sqrt{12,5}$

c) $\sqrt{20}-\sqrt{45}+3\sqrt{18}+\sqrt{72}$

d) $\sqrt{20}-\sqrt{45}+3\sqrt{18}+\sqrt{72}$

e) $\left(\sqrt{6}+\sqrt{5}\right)^2-\sqrt{120}$

f) $\sqrt{72}-\sqrt{5\frac{1}{3}}+4,5\sqrt{2\frac{2}{3}}+2\sqrt{27}$

g) $\left(\sqrt{28}-2\sqrt{3}+\sqrt{7}\right)\sqrt{7}+\sqrt{84}$

h) $\frac{1}{2}\sqrt{48}-2\sqrt{75}-\sqrt{54}+5\sqrt{1\frac{1}{3}}$

HD:

a) $5\sqrt{\frac{1}{5}}+\frac{1}{2}\sqrt{20}+\sqrt{5}=\sqrt{5}+\sqrt{5}+\sqrt{5}=3\sqrt{5}$

b) $\sqrt{\frac{1}{2}}+\sqrt{4,5}+\sqrt{12,5}=\frac{\sqrt{2}}{2}+\frac{3\sqrt{2}}{2}+\frac{5\sqrt{2}}{2}=\frac{9\sqrt{2}}{2}$

c) $\sqrt{20}-\sqrt{45}+3\sqrt{18}+\sqrt{72}=2\sqrt{5}-3\sqrt{5}+9\sqrt{2}+6\sqrt{2}=-\sqrt{5}+15\sqrt{2}$

d) $\sqrt{20}-\sqrt{45}+3\sqrt{18}+\sqrt{72}=2\sqrt{5}-3\sqrt{5}+9\sqrt{2}+6\sqrt{2}=-\sqrt{5}+15\sqrt{2}$

e) $\left(\sqrt{6}+\sqrt{5}\right)^2-\sqrt{120}=6+5+2\sqrt{30}-2\sqrt{30}=11$

f) $\sqrt{72}-\sqrt{5\frac{1}{3}}+4,5\sqrt{2\frac{2}{3}}+2\sqrt{27}=6\sqrt{2}-\frac{4}{3}\sqrt{3}+\sqrt{6}+6\sqrt{3}=6\sqrt{2}+\sqrt{6}+\frac{14}{3}\sqrt{3}$

g) $\left(\sqrt{28}-2\sqrt{3}+\sqrt{7}\right).\sqrt{7}+\sqrt{84}=14-2\sqrt{21}+7+2\sqrt{21}=21$

h) $\frac{1}{2}\sqrt{48}-2\sqrt{75}-\sqrt{54}+5\sqrt{1\frac{1}{3}}=2\sqrt{3}-10\sqrt{3}-3\sqrt{6}+\frac{10}{3}\sqrt{3}=-\frac{14}{3}\sqrt{3}-3\sqrt{6}$

Bài 4:Rút gọn các biểu thức sau

a. $A=\sqrt{200}-\sqrt{32}+\sqrt{72}$

b. $B=4\sqrt{20}-3\sqrt{125}+5\sqrt{45}-15\sqrt{\frac{1}{5}}$

c. $C=(2\sqrt{8}+3\sqrt{5}-7\sqrt{2})(\sqrt{72}-5\sqrt{20}-2\sqrt{2})$

d. $D=\sqrt{5}-\sqrt{3-\sqrt{29-12\sqrt{5}}}=\sqrt{5}-\sqrt{3-\sqrt{(\sqrt{20}-3)^2}}$

e. $E=\sqrt{\sqrt{5}-\sqrt{3-\sqrt{29-6\sqrt{20}}}}$

f. $F=\sqrt{6+2\sqrt{5-\sqrt{13+\sqrt{48}}}}$

g. $G=\sqrt{4+\sqrt{5\sqrt{3}+\sqrt{48-10\sqrt{7+4\sqrt{3}}}}}$

HD:

a. $A=\sqrt{200}-\sqrt{32}+\sqrt{72}=10\sqrt{2}-4\sqrt{2}+6\sqrt{2}=12\sqrt{2}$

b. $B=4\sqrt{20}-3\sqrt{125}+5\sqrt{45}-15\sqrt{\frac{1}{5}}=4.2\sqrt{5}-3.5\sqrt{5}+5.3\sqrt{5}-3\sqrt{\frac{25}{5}}$
$=8\sqrt{5}-15\sqrt{5}-3\sqrt{5}+15\sqrt{5}=5\sqrt{5}$

c. $C=(2\sqrt{8}+3\sqrt{5}-7\sqrt{2})(\sqrt{72}-5\sqrt{20}-2\sqrt{2})=(2.2\sqrt{2}+3\sqrt{5}-7\sqrt{2})(6\sqrt{2}-10\sqrt{5}-2\sqrt{2})$
$=(3\sqrt{5}-3\sqrt{2})(4\sqrt{2}-10\sqrt{5})$

$C=3.2(\sqrt{5}-\sqrt{2})(2\sqrt{2}-5\sqrt{5})=6(2\sqrt{10}-4-25+5\sqrt{10})=6(7\sqrt{10}-29)$

d. $D=\sqrt{5}-\sqrt{3-\sqrt{29-12\sqrt{5}}}=\sqrt{5}-\sqrt{3-\sqrt{(\sqrt{20}-3)^2}}=\sqrt{5}-\sqrt{3-(\sqrt{20}-3)}=\sqrt{5}-\sqrt{6-\sqrt{20}}$

$D=\sqrt{5}-\sqrt{(\sqrt{5}-1)^2}=\sqrt{5}-(\sqrt{5}-1)=1$

e. $E=\sqrt{\sqrt{5}-\sqrt{3-\sqrt{29-6\sqrt{20}}}}=\sqrt{\sqrt{5}-\sqrt{3-\sqrt{(2\sqrt{5}-3)^2}}}=\sqrt{\sqrt{5}-\sqrt{(\sqrt{5}-1)^2}}=\sqrt{\sqrt{5}-\sqrt{5}+1}=1$

f. $F=\sqrt{6+2\sqrt{5-\sqrt{13+\sqrt{48}}}}=\sqrt{6+2\sqrt{5-\sqrt{(2\sqrt{3}+1)^2}}}=\sqrt{6+2\sqrt{4-2\sqrt{3}}}$
$=\sqrt{6+2\sqrt{(\sqrt{3}-1)^2}}=\sqrt{4+2\sqrt{3}}=\sqrt{3}+1$

g. $G=\sqrt{4+\sqrt{5\sqrt{3}+\sqrt{48-10\sqrt{7+4\sqrt{3}}}}}=\sqrt{4+\sqrt{5\sqrt{3}+5\sqrt{48-10(2+\sqrt{3})}}}$
$=\sqrt{4+\sqrt{5\sqrt{3}+5(5-\sqrt{3})}}=\sqrt{9}=3$

Bài 5: Rút gọn các biểu thức sau

a. $A=\dfrac{2}{a-2}.\sqrt{2a^8(a^2-4a+4)}(a\neq 2)$

b. $B=4\sqrt{25u}-\dfrac{15}{2}\sqrt{\dfrac{16u}{9}}-\dfrac{2}{u}\sqrt{\dfrac{169u^3}{4}}(u>0)$

c. $C=5\sqrt{4x}-3\sqrt{\dfrac{100x}{9}}-\dfrac{4}{x}\sqrt{\dfrac{x^3}{4}}(x>0)$

d. $D=-\sqrt{36b}-\dfrac{1}{3}\sqrt{54b}+\dfrac{1}{5}\sqrt{150b}(b\geq 0)$

e) $E=\dfrac{1}{3}\sqrt{9+6v+v^2}+\dfrac{4v}{3}+5\left(v\leq -3\right)$

f) $F=\dfrac{t}{2}+\dfrac{3}{2}\sqrt{4-4t+t^2}-2\left(t\leq 2\right)$

HD:

a. $A=\dfrac{2}{a-2}.\sqrt{2a^8(a^2-4a+4)}=\dfrac{2}{a-2}.\sqrt{2a^8(a-2)^2}=\dfrac{2\sqrt{2}a^4|a-2|}{a-2}$

+) Nếu $a-2>0\Rightarrow A=2\sqrt{2}a^4$

+) Nếu $a-2<0\Rightarrow A=-2\sqrt{2}a^4$

b. $B=4\sqrt{25u}-\dfrac{15}{2}\sqrt{\dfrac{16u}{9}}-\dfrac{2}{u}\sqrt{\dfrac{169u^3}{4}}=20\sqrt{u}-10\sqrt{u}-13\sqrt{u}=-3\sqrt{u}(u\geq 0)$

c. $C=5\sqrt{4x}-3\sqrt{\dfrac{100x}{9}}-\dfrac{4}{x}\sqrt{\dfrac{x^3}{4}}=10\sqrt{x}-10\sqrt{x}-2\sqrt{x}=-2\sqrt{x}$

d. $D=-\sqrt{36b}-\dfrac{1}{3}\sqrt{54b}+\dfrac{1}{5}\sqrt{150b}=-6\sqrt{b}-\dfrac{1}{3}.3.\sqrt{6b}+\dfrac{1}{5}.5.\sqrt{6b}$
$=-6\sqrt{b}-\sqrt{6b}+\sqrt{6b}=-6\sqrt{b}(b\geq 0)$

e) $E=\dfrac{1}{3}\sqrt{9+6v+v^2}+\dfrac{4v}{3}+5\left(v\leq -3\right)=\dfrac{1}{3}\sqrt{\left(v+3\right)^2}+\dfrac{4v}{3}+5=v+4$

f) $F=\dfrac{t}{2}+\dfrac{3}{2}\sqrt{4-4t+t^2}-2\left(t\leq 2\right)=\dfrac{t}{2}+\dfrac{3}{2}|2-t|-2=1-t\left(t\leq 2\right)$

Bài 6: Rút gọn các biểu thức sau (biết a > 0, b > 0):

a) $5\sqrt{a}-3\sqrt{25a^3}+2\sqrt{36ab^2}-2\sqrt{9a}$

b) $\sqrt{64ab^3}-3\sqrt{12a^3b^3}+2ab\sqrt{9ab}-5b\sqrt{81a^3b}$

c) $2\sqrt{3a}-\sqrt{75a}+a\sqrt{\frac{13,5}{2a}}-\frac{2}{5}\sqrt{300a^3}$

HD:

a) $5\sqrt{a}-3\sqrt{25a^3}+2\sqrt{36ab^2}-2\sqrt{9a}=5\sqrt{a}-15a\sqrt{a}+12b\sqrt{a}-6\sqrt{a}=-\sqrt{a}-15a\sqrt{a}+12b\sqrt{a}$

b) $\sqrt{64ab^3}-3\sqrt{12a^3b^3}+2ab\sqrt{9ab}-5b\sqrt{81a^3b}$

$=8b\sqrt{ab}-6ab\sqrt{3ab}+6ab\sqrt{ab}-45ab\sqrt{ab}=8b\sqrt{ab}-6ab\sqrt{3ab}-39ab\sqrt{ab}$

c) $2\sqrt{3a}-\sqrt{75a}+a\sqrt{\frac{13.5}{2a}}-\frac{2}{5}\sqrt{300a^3}=2\sqrt{3a}-5\sqrt{3a}+3\sqrt{3a}-4a\sqrt{3a}=-4a\sqrt{3a}$

Bài 7: Giải các phương trình sau

a. $25\sqrt{\frac{a-3}{25}}-7\sqrt{\frac{4a-12}{9}}-7\sqrt{a^2-9}+18\sqrt{\frac{9a^2-81}{81}}=0$ b. $\sqrt{18x+9}-\sqrt{8x+4}+\frac{1}{3}\sqrt{2x+1}=4$

HD:

a. $25\sqrt{\frac{a-3}{25}}-7\sqrt{\frac{4a-12}{9}}-7\sqrt{a^2-9}+18\sqrt{\frac{9a^2-81}{81}}=0\Leftrightarrow\frac{1}{3}\sqrt{a-3}-\sqrt{a^2-9}=0$

Cách 1: $\frac{1}{3}\sqrt{a-3}-\sqrt{a^2-9}=0\Leftrightarrow\sqrt{a-3}=3\sqrt{a^2-9}\Leftrightarrow\begin{cases}a\geq 3\\a-3=9(a^2-9)\end{cases}\Leftrightarrow a=3$

Cách 2: Điều kiện a ≥ 3

$\frac{1}{3}\sqrt{a-3}-\sqrt{a^2-9}=0\Leftrightarrow\sqrt{a-3}(\frac{1}{3}-\sqrt{a+3})=0\Leftrightarrow\left[\begin{array}{l}a=3(tm)\\a=\frac{-26}{9}(loai)\end{array}\right.$

b. $\sqrt{18x+9}-\sqrt{8x+4}+\frac{1}{3}\sqrt{2x+1}=4\Leftrightarrow\frac{2}{3}\sqrt{2x+1}=4\Leftrightarrow x=\frac{35}{2}$

Dạng 4: Khử mẫu của biểu thức dưới dấu căn bậc hai

Phương pháp giải: Nắm vững cách khử mẫu của biểu thức dưới dấu căn bậc hai

1. Khử mẫu của biểu thức lấy căn

$$\sqrt{\frac{A}{B}}=\sqrt{\frac{AB}{B^2}}=\frac{1}{|B|}\sqrt{AB}(B\neq 0;AB\geq 0)$$

Bài 1: Khử mẫu của các biểu thức dưới dấu căn (giả thiết rằng các biểu thức đã cho có nghĩa):

a) $\sqrt{\frac{1}{600}}$; $\sqrt{\frac{11}{540}}$; $\sqrt{\frac{3}{50}}$; $\sqrt{\frac{5}{98}}$; $\sqrt{\frac{(1-\sqrt{3})^2}{27}}$

b) $ab\sqrt{\frac{a}{b}}$; $\frac{a}{b}\sqrt{\frac{b}{a}}$; $\sqrt{\frac{1}{b}+\frac{1}{b^2}}$; $\sqrt{\frac{9a^3}{36b}}$; $3xy\sqrt{\frac{2}{xy}}$

c) $\sqrt{\frac{2}{3}}$; $\sqrt{\frac{x^2}{5}}$; $\sqrt{\frac{3}{x}}$; $\sqrt{x^2-\frac{x^2}{7}}$; $3xy\sqrt{\frac{2}{xy}}$

HD:

a) $\sqrt{\frac{1}{600}}=\frac{\sqrt{600}}{600}=\frac{10\sqrt{6}}{600}=\frac{\sqrt{6}}{60}$; $\sqrt{\frac{11}{540}}=\frac{\sqrt{11.540}}{540}=\frac{6\sqrt{165}}{540}=\frac{\sqrt{165}}{90}$;

$\sqrt{\frac{3}{50}}=\frac{\sqrt{3.50}}{50}=\frac{5\sqrt{6}}{50}=\frac{\sqrt{6}}{10}$; $\sqrt{\frac{5}{98}}=\frac{\sqrt{5.98}}{98}=\frac{7\sqrt{10}}{98}=\frac{\sqrt{10}}{14}$;

$\sqrt{\frac{(1-\sqrt{3})^2}{27}}=\frac{\left(\sqrt{3}-1\right)\sqrt{27}}{27}=\frac{3\sqrt{3}\left(\sqrt{3}-1\right)}{27}=\frac{9-3\sqrt{3}}{27}=\frac{3-\sqrt{3}}{9}$

b) $ab\sqrt{\frac{a}{b}}=ab\frac{\sqrt{ab}}{b}=a\sqrt{ab}$; $\frac{a}{b}\sqrt{\frac{b}{a}}=\frac{a}{b}\frac{\sqrt{ba}}{a}=\frac{\sqrt{ab}}{b}$;

$\sqrt{\frac{1}{b}+\frac{1}{b^2}}=\sqrt{\frac{b+1}{b^2}}=\frac{\sqrt{b+1}}{b}$; $\sqrt{\frac{9a^3}{36b}}=\frac{3a}{6}\frac{\sqrt{ab}}{b}=\frac{a}{2b}\sqrt{ab}$;

c) $\sqrt{\frac{2}{3}}=\frac{\sqrt{6}}{3}$; $\sqrt{\frac{x^2}{5}}=\frac{x\sqrt{5}}{5}$; $\sqrt{\frac{3}{x}}=\frac{\sqrt{3x}}{x}$;

$\sqrt{x^2-\frac{x^2}{7}}=\sqrt{\frac{7x^2-x^2}{7}}=\sqrt{\frac{6x^2}{7}}=\frac{x}{7}\sqrt{42}$; $3xy\sqrt{\frac{2}{xy}}=3xy\frac{\sqrt{2xy}}{xy}=3\sqrt{2xy}$

Bài 2:Khử mẫu của mỗi biểu thức dưới dấu căn bậc hai sau

a. $\sqrt{\frac{5x^3}{49y}}(x\geq 0;y>0)$

b. $7xy\sqrt{\frac{-3}{xy}}(x<0;y>0)$

c. $\sqrt{\frac{5b}{49a^3}}(a>0,b\geq 0)$

d. $\frac{-1}{4}ab\sqrt{\frac{16}{ab}}(a<0,b<0)$

HD:

a) $\sqrt{\frac{5x^3}{49y}}\left(x\ge 0;y>0\right)=\frac{x}{7}\sqrt{\frac{5x}{y}}=\frac{x}{7}\sqrt{\frac{5xy}{y^2}}=\frac{x}{7|y|}\sqrt{5xy}=\frac{x}{7y}\sqrt{5xy}\left(x\ge 0;y>0\right)$

b) $7xy\sqrt{\frac{-3}{xy}}\left(x<0;y>0\right)=7xy\sqrt{\frac{-3xy}{x^2y^2}}=\frac{7xy}{|xy|}\sqrt{-3xy}=-7\sqrt{-3xy}\left(x<0;y>0\right)$

a) $\sqrt{\frac{5b}{49a^3}}\left(a>0,b\ge 0\right)=\frac{1}{7a}\sqrt{\frac{5b}{a}}=\frac{1}{7a}\sqrt{\frac{5ab}{a^2}}=\frac{1}{7a^2}\sqrt{5ab}\left(a>0,b\ge 0\right)$

b) $\frac{-1}{4}ab\sqrt{\frac{16}{ab}}\left(a<0,b<0\right)=-ab\sqrt{\frac{1}{ab}}=-ab\sqrt{\frac{ab}{a^2b^2}}=-\sqrt{ab}$

Dạng 5: Trục căn thức ở mẫu

Phương pháp giải: Nắm vững cách trục căn thức ở mẫu

1) $\frac{A}{\sqrt{B}}=\frac{A.\sqrt{B}}{B}$ 2) $\frac{m}{\sqrt{A}}$ 3) $\frac{m}{\sqrt{A}-\sqrt{B}}=\frac{m\left(\sqrt{A}+\sqrt{B}\right)}{A-B}$

Bài 1: Trục căn thức ở mẫu của các biểu thức sau (giả thiết rằng các biểu thức đã cho có nghĩa):

a) $\frac{5}{\sqrt{10}}$; $\frac{1}{3\sqrt{3}}$; $\frac{5}{2\sqrt{5}}$; $\frac{2\sqrt{2}+2}{5\sqrt{2}}$; $\frac{y+b\sqrt{y}}{b\sqrt{y}}$

b) $\frac{3}{\sqrt{3}+1}$; $\frac{2}{\sqrt{3}-1}$; $\frac{2+\sqrt{3}}{2-\sqrt{3}}$; $\frac{b}{3+\sqrt{b}}$; $\frac{p}{2\sqrt{p}-1}$

c) $\frac{3}{\sqrt{3}+1}$; $\frac{3}{\sqrt{10}+\sqrt{7}}$; $\frac{1}{\sqrt{x}+\sqrt{y}}$; $\frac{2ab}{\sqrt{a}+\sqrt{b}}$.

d) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{2}}$; $\frac{26}{5-2\sqrt{3}}$; $\frac{2\sqrt{10}-5}{4-\sqrt{10}}$; $\frac{9-2\sqrt{3}}{3\sqrt{6}-2\sqrt{2}}$.

e) $\frac{1}{\sqrt{3}+\sqrt{2}+1}$; $\frac{1}{\sqrt{5}-\sqrt{3}+2}$.

HD:

a) $\frac{5}{\sqrt{10}}=\frac{5\sqrt{10}}{10}=\frac{\sqrt{10}}{2}$; $\frac{1}{3\sqrt{3}}=\frac{\sqrt{3}}{9}$; $\frac{5}{2\sqrt{5}}=\frac{5\sqrt{5}}{10}=\frac{\sqrt{5}}{2}$;

$\frac{2\sqrt{2}+2}{5\sqrt{2}}=\frac{\sqrt{2}\left(2+\sqrt{2}\right)}{5\sqrt{2}}=\frac{2+\sqrt{2}}{5}$; $\frac{y+b\sqrt{y}}{b\sqrt{y}}=\frac{\sqrt{y}\left(\sqrt{y}+b\right)}{b\sqrt{y}}=\frac{\sqrt{y}+b}{b}$

b) $\frac{3}{\sqrt{3}+1}=\frac{3\left(\sqrt{3}-1\right)}{3-1}=\frac{3\left(\sqrt{3}-1\right)}{2}$; $\frac{2}{\sqrt{3}-1}=\frac{2\left(\sqrt{3}+1\right)}{2}=\sqrt{3}+1$;

$\frac{2+\sqrt{3}}{2-\sqrt{3}}=\frac{\left(2+\sqrt{3}\right)\left(2+\sqrt{3}\right)}{1}=7+2\sqrt{3}$; $\frac{b}{3+\sqrt{b}}=\frac{b\left(3-\sqrt{b}\right)}{9-b}$;

$\frac{p}{2\sqrt{p}-1}=\frac{p\left(2\sqrt{p}+1\right)}{4p-1}$

c) $\frac{3}{\sqrt{3}+1}=\frac{3\left(\sqrt{3}-1\right)}{2}$; $\frac{3}{\sqrt{10}+\sqrt{7}}=\frac{3\left(\sqrt{10}-\sqrt{7}\right)}{3}=\sqrt{10}-\sqrt{7}$;

$\frac{1}{\sqrt{x}+\sqrt{y}}=\frac{\sqrt{x}-\sqrt{y}}{x-y}$; $\frac{2ab}{\sqrt{a}+\sqrt{b}}=\frac{2ab\left(\sqrt{a}-\sqrt{b}\right)}{a-b}$.

d) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{2}}=\frac{\sqrt{2}\left(\sqrt{5}-\sqrt{3}\right)}{2}=\frac{\sqrt{10}-\sqrt{6}}{2}$; $\frac{26}{5-2\sqrt{3}}=\frac{26\left(5+2\sqrt{3}\right)}{25-12}=2\left(5+2\sqrt{3}\right)$;

$\frac{2\sqrt{10}-5}{4-\sqrt{10}}=\frac{\left(2\sqrt{10}-5\right)\left(4+\sqrt{10}\right)}{6}=\frac{8\sqrt{10}+20-20-5\sqrt{10}}{6}=\frac{3\sqrt{10}}{6}=\frac{\sqrt{10}}{2}$;

$\frac{9-2\sqrt{3}}{3\sqrt{6}-2\sqrt{2}}=\frac{\left(9-2\sqrt{3}\right)\left(3\sqrt{6}+2\sqrt{2}\right)}{54-8}=\frac{27\sqrt{6}+18\sqrt{2}-6\sqrt{18}-4\sqrt{6}}{46}=\frac{23\sqrt{6}+18\sqrt{2}-18\sqrt{2}}{46}=\frac{\sqrt{6}}{2}$.

e)

$\frac{1}{\sqrt{3}+\sqrt{2}+1}=\frac{\sqrt{3}+\sqrt{2}-1}{\left(\sqrt{3}+\sqrt{2}\right)^2-1}=\frac{\sqrt{3}+\sqrt{2}-1}{5+2\sqrt{6}-1}=\frac{\sqrt{3}+\sqrt{2}-1}{4-2\sqrt{6}}=\frac{\left(\sqrt{3}+\sqrt{2}-1\right)\left(4-2\sqrt{6}\right)}{16-24}$

$=\frac{4\sqrt{3}-2\sqrt{18}+4\sqrt{2}-2\sqrt{12}-4+2\sqrt{6}}{-8}=\frac{4\sqrt{3}-6\sqrt{2}+4\sqrt{2}-4\sqrt{3}-4+2\sqrt{6}}{-8}$;

$=\frac{-2\sqrt{2}-4+2\sqrt{6}}{-8}=\frac{-2\left(\sqrt{2}+2-\sqrt{6}\right)}{-8}=\frac{\sqrt{2}+2-\sqrt{6}}{4}$

$$\frac{1}{\sqrt{5}-\sqrt{3}+2}=\frac{\sqrt{5}-\sqrt{3}-2}{\left(\sqrt{5}-\sqrt{3}\right)^2-4}=\frac{\sqrt{5}-\sqrt{3}-2}{8-2\sqrt{15}-4}=\frac{\sqrt{5}-\sqrt{3}-2}{4-2\sqrt{15}}$$

$$=\frac{\left(\sqrt{5}-\sqrt{3}-2\right)\left(4+2\sqrt{15}\right)}{16-60}=\frac{4\sqrt{5}+10\sqrt{3}-4\sqrt{3}-6\sqrt{5}-8-4\sqrt{15}}{-44}$$

$$=\frac{-2\sqrt{5}+6\sqrt{3}-8-4\sqrt{15}}{-44}=\frac{-2\left(\sqrt{5}-3\sqrt{3}+4+2\sqrt{15}\right)}{-44}=\frac{\sqrt{5}-3\sqrt{3}+4+2\sqrt{15}}{22}$$

Bài 2:Trục căn thức ở mẫu và rút gọn

a. $\frac{1}{2\sqrt{2}-3\sqrt{3}}$ b. $\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}$ c. $\frac{\sqrt{8}}{\sqrt{5}-\sqrt{3}}$ d. $\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}$

HD:

a) $\frac{1}{2\sqrt{2}-3\sqrt{3}}=\frac{1}{\sqrt{8}-\sqrt{27}}=\frac{\sqrt{8}+\sqrt{27}}{8-27}=\frac{\sqrt{8}+\sqrt{27}}{-19}=\frac{-\left(\sqrt{8}+\sqrt{27}\right)}{19}$

b) $\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}=\sqrt{\frac{\left(3-\sqrt{5}\right)\left(3-\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right)}}=\sqrt{\frac{\left(3-\sqrt{5}\right)^2}{4}}=\frac{3-\sqrt{5}}{2}$

c) $\frac{\sqrt{8}}{\sqrt{5}-\sqrt{3}}=\frac{2\sqrt{2}\left(\sqrt{5}+\sqrt{3}\right)}{5-3}=\sqrt{10}+\sqrt{6}$

d) $\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}=\sqrt{\frac{\left(2-\sqrt{3}\right)^2}{2^2-\left(\sqrt{3}\right)^2}}=2-\sqrt{3}$

Bài 3: Rút gọn biểu thức

1) $P=\frac{2}{\sqrt{3}-1}-\sqrt{27}+\frac{3}{\sqrt{3}}$

2) $P=\frac{\sqrt{5}}{\sqrt{5}-2}-2\sqrt{5}$

3) $B=\frac{1}{3+\sqrt{7}}+\frac{1}{3-\sqrt{7}}$

4) $A=\frac{1}{\sqrt{6}-2}+\frac{1}{\sqrt{6}+2}$

5) $A=\frac{1}{2-\sqrt{3}}+\sqrt{7-4\sqrt{3}}$

6) $\left(\frac{\sqrt{21}-\sqrt{7}}{\sqrt{3}-1}+\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}-1}\right):\frac{1}{\sqrt{7}-\sqrt{5}}$

7) $P=(\sqrt{3}-1)\frac{3+\sqrt{3}}{2\sqrt{3}}$

8) $B=\frac{2}{\sqrt{7}-\sqrt{6}}-\sqrt{28}+\sqrt{54}$

9) $A=\frac{4}{\sqrt{3}-1}-\frac{2}{\sqrt{2}+\sqrt{3}}-\sqrt{8}$

10) $\frac{50-\sqrt{25}}{\sqrt{36}}$

11) $A=\frac{5+\sqrt{5}}{\sqrt{5}+2}+\frac{\sqrt{5}}{\sqrt{5}-1}-\frac{3\sqrt{5}}{3+\sqrt{5}}$

12) $P=\frac{\sqrt{4-2\sqrt{3}}}{1-\sqrt{3}}$

13) $M=\frac{6}{2-\sqrt{3}}+\sqrt{(2-\sqrt{3})^2}-\sqrt{75}$

14) $\frac{2}{\sqrt{2}+2}+\frac{1}{3}.\sqrt{18}$

15) $\mathrm{M}=\frac{\sqrt{12}+3}{\sqrt{3}};\ \mathrm{N}=\frac{3-2\sqrt{2}}{\sqrt{2}-1}$

16) $P=(\frac{1}{2-\sqrt{3}}-\frac{1}{2+\sqrt{3}}).\frac{\sqrt{3}-1}{3-\sqrt{3}}$

17) $A=\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}-\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}$

18) $\mathrm{A}=\sqrt{9-4\sqrt{5}}+\frac{1}{\sqrt{5}-2}$

19) $A=\left(\frac{15}{\sqrt{6}+1}+\frac{4}{\sqrt{6}-2}-\frac{12}{3-\sqrt{6}}\right)\left(\sqrt{6}+11\right)$

20) $B=\left(1-\frac{5+\sqrt{5}}{1+\sqrt{5}}\right)\left(\frac{5-\sqrt{5}}{1-\sqrt{5}}-1\right)$

21) $A=\frac{3+2\sqrt{3}}{\sqrt{3}}+\frac{2+\sqrt{2}}{\sqrt{2}+1}-\left(\sqrt{2}+\sqrt{3}\right)$

22) $B=\left(\frac{5-2\sqrt{5}}{2-\sqrt{5}}-2\right)\left(\frac{5+3\sqrt{5}}{3+\sqrt{5}}-2\right)$

23) $A=\frac{\sqrt{14}-\sqrt{7}}{2-\sqrt{2}}$

24) $B=\frac{1}{\sqrt{7-\sqrt{24}}+1}-\frac{1}{\sqrt{7+\sqrt{24}}-1}$

25) $C=(\frac{15}{\sqrt{6}+1}+\frac{4}{\sqrt{6}-2}-\frac{12}{3-\sqrt{6}})(\sqrt{6}+11)$

26) $D=\frac{\sqrt{3}}{\sqrt{\sqrt{3}+1}-1}-\frac{\sqrt{3}}{\sqrt{\sqrt{3}+1}+1}$

HD:

1) $P=\frac{2\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}-3\sqrt{3}+\sqrt{3}=\frac{2\left(\sqrt{3}+1\right)}{3-1}-2\sqrt{3}=\sqrt{3}+1-2\sqrt{3}=1-\sqrt{3}$

2) $=\frac{\sqrt{5}-2\sqrt{5}(\sqrt{5}-2)}{\sqrt{5}-2}=\frac{\sqrt{5}-10+4\sqrt{5}}{\sqrt{5}-2}$

$=\frac{5\sqrt{5}-10}{\sqrt{5}-2}=\frac{5(\sqrt{5}-2)}{\sqrt{5}-2}=5$

3) $B=\frac{1}{3+\sqrt{7}}+\frac{1}{3-\sqrt{7}}=\frac{6}{3^2-\sqrt{7}^2}=\frac{6}{9-7}=3$

4) $A=\frac{1}{\sqrt{6}-2}+\frac{1}{\sqrt{6}+2}=\frac{\sqrt{6}+2+\sqrt{6}-2}{(\sqrt{6}-2)(\sqrt{6}+2)}=\frac{2\sqrt{6}}{6-4}=\sqrt{6}$

5) $=\frac{1}{2-\sqrt{3}}+\sqrt{4-4\sqrt{3}+3}=\frac{1}{2-\sqrt{3}}+\sqrt{(2-\sqrt{3})^2}$

$=\frac{1}{2-\sqrt{3}}+2-\sqrt{3}=\frac{2+\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})}+2-\sqrt{3}=\frac{2+\sqrt{3}}{1}+2-\sqrt{3}=4$

6) $A=\left[\frac{\sqrt{7}(\sqrt{3}-1)}{\sqrt{3}-1}+\frac{\sqrt{5}(\sqrt{2}-1)}{\sqrt{2}-1}\right](\sqrt{7}-\sqrt{5})$

$A=(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})=7-5=2$

7) $P=(\sqrt{3}-1)\dfrac{3+\sqrt{3}}{2\sqrt{3}}=(\sqrt{3}-1)\dfrac{\sqrt{3}(\sqrt{3}+1)}{2\sqrt{3}}=\dfrac{(\sqrt{3}-1)(\sqrt{3}+1)}{2}=\dfrac{3-1}{2}=1$

8) $B=\dfrac{2}{\sqrt{7}-\sqrt{6}}-\sqrt{28}+\sqrt{54}=\dfrac{2(\sqrt{7}+\sqrt{6})}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}-\sqrt{7.4}+\sqrt{9.6}$

$=\dfrac{2\sqrt{7}+2\sqrt{6}}{7-6}-2\sqrt{7}+3\sqrt{6}=2\sqrt{7}+2\sqrt{6}-2\sqrt{7}+3\sqrt{6}=5\sqrt{6}$

9) $=\dfrac{4(\sqrt{3}+1)}{3-1}-\dfrac{2(\sqrt{3}-\sqrt{2})}{3-2}-2\sqrt{2}=2\sqrt{3}+x-2\sqrt{3}+2\sqrt{2}-2\sqrt{2}=2$

10) 1. $\dfrac{50-\sqrt{25}}{\sqrt{36}}=\dfrac{50-5}{6}=\dfrac{15}{2}$

$A=\dfrac{5+\sqrt{5}}{\sqrt{5}+2}+\dfrac{\sqrt{5}}{\sqrt{5}-1}-\dfrac{3\sqrt{5}}{3+\sqrt{5}}$

11) $=\dfrac{(5+\sqrt{5})(\sqrt{5}-2)}{(\sqrt{5}-2)(\sqrt{5}+2)}+\dfrac{\sqrt{5}(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)}-\dfrac{3\sqrt{5}(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$

$=3\sqrt{5}-5+\dfrac{5+\sqrt{5}}{4}-\dfrac{9\sqrt{5}-15}{4}$

$=3\sqrt{5}-5+\dfrac{5+\sqrt{5}-9\sqrt{5}+15}{4}=3\sqrt{5}-5+5-2\sqrt{5}=\sqrt{5}$

12) $P=\dfrac{\sqrt{4-2\sqrt{3}}}{1-\sqrt{3}}=\dfrac{\sqrt{(\sqrt{3}-1)^2}}{1-\sqrt{3}}=\dfrac{|\sqrt{3}-1|}{1-\sqrt{3}}=-1$

13) $M=\dfrac{6}{2-\sqrt{3}}+|2-\sqrt{3}|-\sqrt{75}$

$=6(2+\sqrt{3})+2-\sqrt{3}-5\sqrt{3}=14$

14) $\dfrac{2}{\sqrt{2}+2}+\dfrac{1}{3}\sqrt{18}=\dfrac{\sqrt{2}}{1+\sqrt{2}}+\dfrac{\sqrt{9.2}}{3}=\dfrac{\sqrt{2}(\sqrt{2}-1)}{1-2}+\dfrac{3\sqrt{2}}{3}$

$=\dfrac{\sqrt{2}-2}{-1}+\sqrt{2}=2-\sqrt{2}+\sqrt{2}=2$

15) $\mathrm{M}=\dfrac{\sqrt{12}+3}{\sqrt{3}}=\dfrac{\sqrt{3}\left(2+\sqrt{3}\right)}{\sqrt{3}}=2+\sqrt{3};$

$\mathrm{N}=\dfrac{3-2\sqrt{2}}{\sqrt{2}-1}=\dfrac{\left(3-2\sqrt{2}\right)\left(\sqrt{2}+1\right)}{\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)}=\dfrac{3\sqrt{2}+3-2\sqrt{2}\sqrt{2}-2\sqrt{2}}{\left(\sqrt{2}\right)^2-1^2}=\dfrac{\sqrt{2}+3-4}{2-1}=\sqrt{2}-1;$

17) $A=\sqrt{\dfrac{2+\sqrt{3}}{2-\sqrt{3}}}-\sqrt{\dfrac{2-\sqrt{3}}{2+\sqrt{3}}}=\dfrac{2+\sqrt{3}}{4-3}-\dfrac{2-\sqrt{3}}{4-3}=2\sqrt{3}.$

$$\sqrt{9-4\sqrt{5}}+\frac{1}{\sqrt{5}-2}=\sqrt{\left(2-\sqrt{5}\right)^2}+\frac{\sqrt{5}+2}{\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)}$$

18) $=\left|2-\sqrt{5}\right|+\frac{\sqrt{5}+2}{5-4}=\sqrt{5}-2+\sqrt{5}+2=2\sqrt{5}$

19) Ta có: $\frac{15}{\sqrt{6}+1}=\frac{15\left(\sqrt{6}-1\right)}{6-1}=3\left(\sqrt{6}-1\right);\frac{4}{\sqrt{6}-2}=2\left(\sqrt{6}+2\right);\frac{12}{3-\sqrt{6}}=4\left(3+\sqrt{6}\right)\rightarrow A=-115$

20) Ta có: $\frac{5+\sqrt{5}}{1+\sqrt{5}}=\sqrt{5};\frac{5-\sqrt{5}}{1-\sqrt{5}}=-\sqrt{5}\rightarrow B=4$

21) $A=\frac{3+2\sqrt{3}}{\sqrt{3}}+\frac{2+\sqrt{2}}{\sqrt{2}+1}-\left(\sqrt{2}+\sqrt{3}\right)\rightarrow A=2$

22) $B=\left(\frac{5-2\sqrt{5}}{2-\sqrt{5}}-2\right)\left(\frac{5+3\sqrt{5}}{3+\sqrt{5}}-2\right)\rightarrow B=-1$

23) $A=\frac{\sqrt{14}-\sqrt{7}}{2-\sqrt{2}}=\frac{\sqrt{7}(\sqrt{2}-1)}{\sqrt{2}(\sqrt{2}-1)}=\frac{\sqrt{7}}{\sqrt{2}}=\frac{\sqrt{7}.\sqrt{2}}{\sqrt{2}.\sqrt{2}}=\frac{\sqrt{14}}{2}$

$$B=\frac{1}{\sqrt{7-\sqrt{24}}+1}-\frac{1}{\sqrt{7+\sqrt{24}}-1}=\frac{1}{\sqrt{7-2\sqrt{6}}+1}-\frac{1}{\sqrt{7+2\sqrt{6}}-1}$$

24) $=\frac{1}{\sqrt{(\sqrt{6}-1)^2}+1}-\frac{1}{\sqrt{(\sqrt{6}+1)^2}-1}$

$=\frac{1}{\sqrt{6}-1+1}-\frac{1}{\sqrt{6}+1-1}=0$

$C=(\frac{15}{\sqrt{6}+1}+\frac{4}{\sqrt{6}-2}-\frac{12}{3-\sqrt{6}})(\sqrt{6}+11)$

25)

$=[\frac{15(\sqrt{6}-1)}{6-1}+\frac{4(\sqrt{6}+2)}{6-2}+\frac{12(3+\sqrt{6})}{9-6}](\sqrt{6}+11)=-115$

26) $D=\frac{\sqrt{3}}{\sqrt{\sqrt{3}+1}-1}-\frac{\sqrt{3}}{\sqrt{\sqrt{3}+1}+1}=\sqrt{3}[\frac{(\sqrt{\sqrt{3}+1}+1)-(\sqrt{\sqrt{3}+1}-1)}{(\sqrt{\sqrt{3}+1}-1)(\sqrt{\sqrt{3}+1}+1)}=\sqrt{3}.\frac{2}{\sqrt{3}+1-1}=2$

Bài 4: Rút gọn biểu thức

a. $\frac{1}{1-\sqrt{3}}-\frac{1}{1+\sqrt{3}}$

b. $\frac{3}{\sqrt{5}-\sqrt{2}}+\frac{4}{\sqrt{6}+\sqrt{2}}+\frac{1}{\sqrt{6}+\sqrt{5}}$

c. $(\frac{2}{\sqrt{5}-\sqrt{3}}-\frac{2}{\sqrt{5}+\sqrt{3}}):\frac{2+\sqrt{3}}{\sqrt{3}-2}$ d. $D=\sqrt{\frac{3\sqrt{3}-4}{2\sqrt{3}+1}}-\sqrt{\frac{\sqrt{3}+4}{5-2\sqrt{3}}}$

HD:

a. $\frac{1}{1-\sqrt{3}}-\frac{1}{1+\sqrt{3}}=\frac{1+\sqrt{3}}{(1-\sqrt{3})(1+\sqrt{3})}-\frac{1-\sqrt{3}}{(1-\sqrt{3})(1+\sqrt{3})}=\frac{1+\sqrt{3}}{1-3}-\frac{1-\sqrt{3}}{1-3}=-\sqrt{3}$

b. $B=\frac{3(\sqrt{5}+\sqrt{2})}{3}+\frac{4(\sqrt{6}-\sqrt{2})}{4}+(\sqrt{6}-\sqrt{5})=2\sqrt{6}$

c. $(\frac{2}{\sqrt{5}-\sqrt{3}}-\frac{2}{\sqrt{5}+\sqrt{3}}):\frac{2+\sqrt{3}}{\sqrt{3}-2}=\left[\frac{2(\sqrt{5}+\sqrt{3})}{2}-\frac{2(\sqrt{5}-\sqrt{3})}{2}\right]:\frac{2+\sqrt{3}}{\sqrt{3}-2}$

$=(\sqrt{5}+\sqrt{3}-\sqrt{5}+\sqrt{3}):\frac{2+\sqrt{3}}{\sqrt{3}-2}=2\sqrt{3}:\frac{2+\sqrt{3}}{\sqrt{3}-2}=\frac{2\sqrt{3}(\sqrt{3}-2)}{\sqrt{3}+2}=\frac{6-4\sqrt{3}}{\sqrt{3}+2}=2\sqrt{3}(\sqrt{3}-2)^2$

d. $D=\sqrt{\frac{3\sqrt{3}-4}{2\sqrt{3}+1}}-\sqrt{\frac{\sqrt{3}+4}{5-2\sqrt{3}}}=\sqrt{\frac{(3\sqrt{3}-4)(2\sqrt{3}-1)}{(2\sqrt{3})^2-1}}-\sqrt{\frac{(\sqrt{3}+4)(5+2\sqrt{3})}{5^2-(2\sqrt{3})^2}}$

$=\sqrt{\frac{22-11\sqrt{3}}{11}}-\sqrt{\frac{26+13\sqrt{3}}{13}}=\sqrt{2-\sqrt{3}}-\sqrt{2+\sqrt{3}}=\sqrt{\frac{4-2\sqrt{3}}{2}}-\sqrt{\frac{4+2\sqrt{3}}{2}}=\frac{1}{\sqrt{2}}\left(\sqrt{(\sqrt{3}-1)^2}-\sqrt{(\sqrt{3}+1)^2}\right)$

$=-\sqrt{2}$

Bài 5: Rút gọn biểu thức $A=\frac{2\sqrt{8}-\sqrt{12}}{\sqrt{18}-\sqrt{48}}-\frac{\sqrt{5}+\sqrt{27}}{\sqrt{30}+\sqrt{162}}$

HD:

Ta có $A=\frac{2\sqrt{8}-\sqrt{12}}{\sqrt{18}-\sqrt{48}}-\frac{\sqrt{5}+\sqrt{27}}{\sqrt{30}+\sqrt{162}}=\frac{4\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-4\sqrt{3}}-\frac{\sqrt{5}+3\sqrt{3}}{\sqrt{30}+9\sqrt{2}}=\frac{\left(4\sqrt{2}-2\sqrt{3}\right)\left(3\sqrt{2}+4\sqrt{3}\right)}{\left(3\sqrt{2}-4\sqrt{3}\right)\left(3\sqrt{2}+4\sqrt{3}\right)}$

$-\frac{\left(\sqrt{5}+3\sqrt{3}\right)\left(\sqrt{30}-9\sqrt{2}\right)}{\left(\sqrt{30}+9\sqrt{2}\right)\left(\sqrt{30}-9\sqrt{2}\right)}=\frac{24+16\sqrt{6}-6\sqrt{6}-24}{18-48}-\frac{\sqrt{150}-9\sqrt{10}+3\sqrt{90}-27\sqrt{6}}{30-162}$

$=\frac{-10\sqrt{6}}{30}-\frac{5\sqrt{6}-9\sqrt{10}+9\sqrt{10}-27\sqrt{6}}{-132}=\frac{-\sqrt{6}}{3}+\frac{-22\sqrt{6}}{132}=\frac{-\sqrt{6}}{3}+\frac{-\sqrt{6}}{6}=\frac{-2\sqrt{6}-\sqrt{6}}{6}=\frac{-3\sqrt{6}}{6}=\frac{-\sqrt{6}}{2}.$

Bài 6: Thực hiện các phép tính sau

1) $A=\frac{10+2\sqrt{10}}{\sqrt{5}+\sqrt{2}}+\frac{8}{1-\sqrt{5}}$

2) $B=\frac{1}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\frac{1}{\sqrt{2}-\sqrt{2+\sqrt{3}}}$

3) $C=\frac{\sqrt{3-\sqrt{5}}\left(3+\sqrt{5}\right)}{\sqrt{10}+\sqrt{2}}$

4) $D=\frac{6+4\sqrt{2}}{\sqrt{2}+\sqrt{6+4\sqrt{2}}}+\frac{6-4\sqrt{2}}{\sqrt{2}-\sqrt{6-4\sqrt{2}}}$

5) $E=\dfrac{\left(\sqrt{5}+2\right)^2-8\sqrt{5}}{2\sqrt{5}-4}$

6) $F=\dfrac{4}{\sqrt{3}+1}+\dfrac{1}{\sqrt{3}-2}+\dfrac{6}{\sqrt{3}-3}$

7) $G=\dfrac{2}{3+\sqrt{5}}+\dfrac{2}{3-\sqrt{5}}$

8) $H=\dfrac{\sqrt{3}}{1-\sqrt{\sqrt{3}+1}}+\dfrac{\sqrt{3}}{1+\sqrt{\sqrt{3}+1}}$

HD:

1) $A=\dfrac{10+2\sqrt{10}}{\sqrt{5}+\sqrt{2}}+\dfrac{8}{1-\sqrt{5}}=\dfrac{2\sqrt{5}.\sqrt{5}+2\sqrt{5}.\sqrt{2}}{\sqrt{5}+\sqrt{2}}+\dfrac{8\left(1+\sqrt{5}\right)}{\left(1-\sqrt{5}\right)\left(1+\sqrt{5}\right)}=\dfrac{2\sqrt{5}\left(\sqrt{5}+\sqrt{2}\right)}{\sqrt{5}+\sqrt{2}}+\dfrac{8\left(1+\sqrt{5}\right)}{1-5}$

$\Rightarrow A=2\sqrt{5}-2\left(1+\sqrt{5}\right)\Rightarrow A=-2$

2) $B=\dfrac{1}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\dfrac{1}{\sqrt{2}-\sqrt{2+\sqrt{3}}}=\dfrac{\sqrt{2}-\sqrt{2+\sqrt{3}}+\sqrt{2}+\sqrt{2+\sqrt{3}}}{\left(\sqrt{2}+\sqrt{2+\sqrt{3}}\right)\left(\sqrt{2}-\sqrt{2+\sqrt{3}}\right)}=\dfrac{2\sqrt{2}}{\left(\sqrt{2}\right)^2-\left(\sqrt{2+\sqrt{3}}\right)^2}=\dfrac{2\sqrt{2}}{2-2-\sqrt{3}}$

$=\dfrac{-2\sqrt{2}}{\sqrt{3}}=\dfrac{-2\sqrt{6}}{3}.$

3)

$C=\dfrac{\sqrt{3-\sqrt{5}}\left(3+\sqrt{5}\right)}{\sqrt{10}+\sqrt{2}}=\dfrac{\left(\sqrt{3-\sqrt{5}}.\sqrt{3+\sqrt{5}}\right)\sqrt{3+\sqrt{5}}}{\sqrt{2}.\left(\sqrt{5}+1\right)}=\dfrac{\sqrt{9-5}.\sqrt{3+\sqrt{5}}}{\sqrt{2}.\left(\sqrt{5}+1\right)}=\dfrac{\sqrt{6+2\sqrt{5}}}{\sqrt{5}+1}=\dfrac{\sqrt{\left(\sqrt{5}+1\right)^2}}{\sqrt{5}+1}=\dfrac{\sqrt{5}+1}{\sqrt{5}+1}=1$

4) $D=\dfrac{6+4\sqrt{2}}{\sqrt{2}+\sqrt{6+4\sqrt{2}}}+\dfrac{6-4\sqrt{2}}{\sqrt{2}-\sqrt{6-4\sqrt{2}}}=\dfrac{4+2.2\sqrt{2}+2}{\sqrt{2}+\sqrt{4+2.2\sqrt{2}+2}}+\dfrac{4-2.2\sqrt{2}+2}{\sqrt{2}-\sqrt{4-2.2\sqrt{2}+2}}=\dfrac{\left(2+\sqrt{2}\right)^2}{\sqrt{2}+\sqrt{\left(2+\sqrt{2}\right)^2}}$

$+\dfrac{\left(2-\sqrt{2}\right)^2}{\sqrt{2}-\sqrt{\left(2-\sqrt{2}\right)^2}}=\dfrac{\left(2+\sqrt{2}\right)^2}{2+2\sqrt{2}}+\dfrac{\left(2-\sqrt{2}\right)^2}{2\sqrt{2}-2}=2\sqrt{2}$

5) $E=\dfrac{\left(\sqrt{5}+2\right)^2-8\sqrt{5}}{2\sqrt{5}-4}=\dfrac{5+4\sqrt{5}+4-8\sqrt{5}}{2\sqrt{5}-4}=\dfrac{\left(2-\sqrt{5}\right)^2}{2\left(\sqrt{5}-2\right)}=\dfrac{\sqrt{5}-2}{2}$

6) $F=\dfrac{4}{\sqrt{3}+1}+\dfrac{1}{\sqrt{3}-2}+\dfrac{6}{\sqrt{3}-3}=\dfrac{42-28\sqrt{3}}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-2\right)\left(\sqrt{3}-3\right)}=\dfrac{42-28\sqrt{3}}{-6+4\sqrt{3}}=-7$

7) $G=\dfrac{2}{3+\sqrt{5}}+\dfrac{2}{3-\sqrt{5}}=\dfrac{2.\left(3-\sqrt{5}\right)+2.\left(3+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right)}=\dfrac{12}{9-5}=\dfrac{12}{4}=3$

8) $H=\dfrac{\sqrt{3}}{1-\sqrt{\sqrt{3}+1}}+\dfrac{\sqrt{3}}{1+\sqrt{\sqrt{3}+1}}=\dfrac{\sqrt{3}+\sqrt{3\sqrt{3}+3}+\sqrt{3}-\sqrt{3\sqrt{3}+3}}{1-\left(\sqrt{\sqrt{3}+1}\right)^2}=-2$

Bài 7: Rút gọn các biểu thức sau

a) $A=\dfrac{2\sqrt{15}-2\sqrt{10}+\sqrt{6}-3}{2\sqrt{5}-2\sqrt{10}-\sqrt{3}+\sqrt{6}}$ b) $B=\dfrac{\sqrt{2}+\sqrt{3}+\sqrt{6}+\sqrt{8}+\sqrt{16}}{\sqrt{2}+\sqrt{3}+\sqrt{4}}$

c) $C=\left(\sqrt{\dfrac{2}{3}}+\sqrt{\dfrac{3}{2}}+2\right)\left(\dfrac{\sqrt{2}+\sqrt{3}}{4\sqrt{2}}+\dfrac{\sqrt{3}}{\sqrt{2}+\sqrt{3}}\right)\left(24+8\sqrt{6}\right)$

HD:

a) $A=\dfrac{2\sqrt{15}-2\sqrt{10}+\sqrt{6}-3}{2\sqrt{5}-2\sqrt{10}-\sqrt{3}+\sqrt{6}}=\dfrac{\left(2\sqrt{15}-2\sqrt{10}\right)+\left(\sqrt{6}-3\right)}{\left(2\sqrt{5}-2\sqrt{10}\right)+\left(\sqrt{6}-\sqrt{3}\right)}=\dfrac{2\sqrt{5}\left(\sqrt{3}-\sqrt{2}\right)+\sqrt{3}\left(\sqrt{2}-\sqrt{3}\right)}{2\sqrt{5}\left(1-\sqrt{2}\right)+\sqrt{3}\left(\sqrt{2}-1\right)}=\dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-1}$

$=\dfrac{\left(\sqrt{2}-\sqrt{3}\right)\left(\sqrt{2}+1\right)}{2-1}=\left(\sqrt{2}-\sqrt{3}\right)\left(\sqrt{2}+1\right)$

b) $B=\dfrac{\sqrt{2}+\sqrt{3}+\sqrt{6}+\sqrt{8}+\sqrt{16}}{\sqrt{2}+\sqrt{3}+\sqrt{4}}=\dfrac{\sqrt{2}+\sqrt{3}+\sqrt{6}+2\sqrt{2}+4}{\sqrt{2}+\sqrt{3}+\sqrt{4}}=\dfrac{\left(\sqrt{2}+\sqrt{3}+2\right)+\left(\sqrt{6}+2\sqrt{2}+2\right)}{\sqrt{2}+\sqrt{3}+2}$

$=\dfrac{\left(\sqrt{2}+\sqrt{3}+2\right)\left(1+\sqrt{2}\right)}{\sqrt{2}+\sqrt{3}+2}=1+\sqrt{2}$

c)

$C=\left(\sqrt{\dfrac{2}{3}}+\sqrt{\dfrac{3}{2}}+2\right)\left(\dfrac{\sqrt{2}+\sqrt{3}}{4\sqrt{2}}+\dfrac{\sqrt{3}}{\sqrt{2}+\sqrt{3}}\right)\left(24+8\sqrt{6}\right)=\left(\dfrac{\sqrt{6}}{3}+\dfrac{\sqrt{6}}{2}+2\right)\left[\dfrac{\left(\sqrt{2}+\sqrt{3}\right)\left(24+8\sqrt{6}\right)}{4\sqrt{2}}-\dfrac{\sqrt{3}\left(24+8\sqrt{6}\right)}{\sqrt{2}+\sqrt{3}}\right]$

$\cdot\left[\dfrac{2-\sqrt{6}-\sqrt{6}-3}{\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{2}-\sqrt{3}\right)}\right]=\dfrac{5\sqrt{6}+12}{6}\cdot\left[\dfrac{\left(\sqrt{2}+\sqrt{3}\right)\left(3\sqrt{2}+2\sqrt{3}\right).4\sqrt{2}}{4\sqrt{2}}-\dfrac{\sqrt{3}.8\sqrt{3}\left(\sqrt{2}+\sqrt{3}\right)}{\sqrt{2}+\sqrt{3}}\right]\cdot\dfrac{-2\sqrt{6}-1}{2-3}$

$=\dfrac{5\sqrt{6}+12}{6}\left[\left(\sqrt{2}+\sqrt{3}\right)\left(2\sqrt{3}+3\sqrt{2}\right)-24\right].\left(2\sqrt{6}+1\right)=\dfrac{5\sqrt{6}+12}{6}\left(5\sqrt{6}-12\right)\left(2\sqrt{6}-1\right)=2\sqrt{6}-1.$

Bài 8: Rút gọn biểu thức $A=\dfrac{1}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\dfrac{1}{\sqrt{2}-\sqrt{2-\sqrt{3}}}$

HD:

$A=\dfrac{1}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\dfrac{1}{\sqrt{2}-\sqrt{2-\sqrt{3}}}=\dfrac{\sqrt{2}-\sqrt{2+\sqrt{3}}}{2-\left(2+\sqrt{3}\right)}+\dfrac{\sqrt{2}+\sqrt{2-\sqrt{3}}}{2-\left(2-\sqrt{3}\right)}=\dfrac{\sqrt{2}-\sqrt{2+\sqrt{3}}}{-\sqrt{3}}+\dfrac{\sqrt{2}+\sqrt{2-\sqrt{3}}}{\sqrt{3}}$

$$=\frac{\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}}{\sqrt{3}}=\frac{\sqrt{\left(1+\sqrt{3}\right)^2}+\sqrt{\left(1-\sqrt{3}\right)^2}}{\sqrt{6}}=\frac{1+\sqrt{3}-1+\sqrt{3}}{\sqrt{6}}=\sqrt{2}$$

Bài 9:Trục căn thức ở mẫu

a. $A=\frac{\sqrt{6}+\sqrt{14}}{2\sqrt{3}-\sqrt{7}}$ b. $B=\frac{3+4\sqrt{3}}{\sqrt{6}+\sqrt{2}-\sqrt{5}}$ c. $C=\frac{1}{2+\sqrt{5}+2\sqrt{2}+\sqrt{10}}$

d. $D=\frac{31}{2+\sqrt{2}-\sqrt{5}}$ e. $E=\frac{1}{\sqrt{10}+\sqrt{15}+\sqrt{14}+\sqrt{21}}$

f. $F=\frac{3+2\sqrt{3}}{\sqrt{3}}+\frac{2+\sqrt{2}}{\sqrt{2}+1}-(\sqrt{2}+\sqrt{3})$

HD:

a. $A=\frac{\sqrt{6}+\sqrt{14}}{2\sqrt{3}-\sqrt{7}}=\frac{\sqrt{2}(\sqrt{3}+\sqrt{7})(2\sqrt{3}+\sqrt{7})}{(2\sqrt{3}-\sqrt{7})(2\sqrt{3}+\sqrt{7})}=\frac{\sqrt{2}(6+2\sqrt{21}+\sqrt{21}+7)}{12-5}=\frac{\sqrt{2}(13+3\sqrt{21})}{5}$

b. $B=\frac{3+4\sqrt{3}}{\sqrt{6}+\sqrt{2}-\sqrt{5}}=\frac{(3+4\sqrt{3})(\sqrt{6}+\sqrt{2}+\sqrt{5})}{(\sqrt{6}+\sqrt{2}-\sqrt{5})(\sqrt{6}+\sqrt{2}+\sqrt{5})}=\frac{(3+4\sqrt{3})(\sqrt{6}+\sqrt{2}+\sqrt{5})}{3+4\sqrt{3}}=\sqrt{6}+\sqrt{2}+\sqrt{5}$

c. $C=\frac{1}{2+\sqrt{5}+2\sqrt{2}+\sqrt{10}}=\frac{1}{(\sqrt{2}+1)(\sqrt{5}+2)}=\frac{(\sqrt{2}-1)(\sqrt{5}-2)}{(2-1)(5-4)}=(\sqrt{2}-1)(\sqrt{5}-2)$

d. $D=\frac{31}{2+\sqrt{2}-\sqrt{5}}=\frac{31(2+\sqrt{2}+\sqrt{5})}{(2+\sqrt{2})^2-5}=\frac{31(2+\sqrt{2}+\sqrt{5})}{1+4\sqrt{2}}=\frac{31(2+\sqrt{2}+\sqrt{5})(4\sqrt{2}-1)}{(4\sqrt{2})^2-1}$

$=(2+\sqrt{2}+\sqrt{5})(4\sqrt{2}-1)$

e. $E=\frac{1}{\sqrt{10}+\sqrt{15}+\sqrt{14}+\sqrt{21}}=\frac{1}{(\sqrt{2}+\sqrt{3})(\sqrt{5}+\sqrt{7})}=\frac{(\sqrt{3}-\sqrt{2})(\sqrt{7}-\sqrt{5})}{2}$

f. $F=\frac{3+2\sqrt{3}}{\sqrt{3}}+\frac{2+\sqrt{2}}{\sqrt{2}+1}-(\sqrt{2}+\sqrt{3})=2$

Bài 10: Thực hiện các phép tính sau:

a) $\frac{13\sqrt{2}-4\sqrt{6}}{24-4\sqrt{3}}$ b) $\frac{\sqrt{3-2\sqrt{2}}}{\sqrt{17-12\sqrt{2}}}$ c) $\frac{9\sqrt{6}-12\sqrt{3}}{3\sqrt{6}-3\sqrt{3}}$

d) $\frac{\sqrt{45}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ e) $\frac{\sqrt{\sqrt{5}+\sqrt{2}}}{\sqrt{3\sqrt{5}-3\sqrt{2}}}$ f) $\frac{3+4\sqrt{3}}{\sqrt{6}+\sqrt{2}-\sqrt{5}}$

HD:

a) $\frac{13\sqrt{2}-4\sqrt{6}}{24-4\sqrt{3}}=\frac{\left(13\sqrt{2}-4\sqrt{6}\right).\left(6+\sqrt{3}\right)}{4.\left(36-3\right)}=\frac{6\sqrt{2}-\sqrt{6}}{12}$

b) $\frac{\sqrt{3-2\sqrt{2}}}{\sqrt{17-12\sqrt{2}}}=\frac{\sqrt{2}-1}{3-2\sqrt{2}}=\frac{\left(\sqrt{2}-1\right).\left(3+2\sqrt{2}\right)}{9-8}=\sqrt{2}+1$

c) $\frac{9\sqrt{6}-12\sqrt{3}}{3\sqrt{6}-3\sqrt{3}}=\frac{3\sqrt{2}-4}{\sqrt{2}-1}=\frac{\left(3\sqrt{2}-4\right).\left(\sqrt{2}+1\right)}{2-1}=2-\sqrt{2}$

d) $\frac{\sqrt{45}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{\left(\sqrt{45}-\sqrt{2}\right).\left(\sqrt{5}+\sqrt{2}\right)}{5-2}=\frac{13+2\sqrt{10}}{3}$

e) $\frac{\sqrt{\sqrt{5}+\sqrt{2}}}{\sqrt{3\sqrt{5}-3\sqrt{2}}}=\frac{\sqrt{\left(\sqrt{5}+\sqrt{2}\right).\left(3\sqrt{5}-3\sqrt{2}\right)}}{3\sqrt{5}-3\sqrt{2}}=\frac{3}{3.\left(\sqrt{5}-\sqrt{2}\right)}=\frac{\sqrt{5}+\sqrt{2}}{3}$

f) $\frac{3+4\sqrt{3}}{\sqrt{6}+\sqrt{2}-\sqrt{5}}=\frac{\left(3+4\sqrt{3}\right).\left(\sqrt{6}+\sqrt{2}-\sqrt{5}\right)}{8+4\sqrt{3}-5}=\frac{7\sqrt{6}+15\sqrt{2}+3\sqrt{5}+4\sqrt{15}}{3+4\sqrt{3}}$

$=\frac{\left(7\sqrt{6}+15\sqrt{2}+3\sqrt{5}+4\sqrt{15}\right).\left(4\sqrt{3}-3\right)}{48-9}=\frac{26\sqrt{2}+13\sqrt{6}+13\sqrt{5}-4\sqrt{15}}{13}$

Bài 11: Thực hiện các phép tính sau:

a) $A=\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2}}$ b) $B=\frac{\sqrt{6+\sqrt{35}}}{\sqrt{2}}$ c) $C=\frac{\sqrt{8-\sqrt{15}}}{\sqrt{30}-\sqrt{2}}$

HD:

a) $A=\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2}}=\sqrt{\frac{4-2\sqrt{3}}{4}}=\frac{\sqrt{3}-1}{2}$

b) $B=\frac{\sqrt{6+\sqrt{35}}}{\sqrt{2}}=\sqrt{\frac{12+2\sqrt{35}}{4}}=\frac{\sqrt{7}+\sqrt{5}}{2}$

c) $C=\frac{\sqrt{8-\sqrt{15}}}{\sqrt{30}-\sqrt{2}}=\frac{1}{\sqrt{15}-1}.\sqrt{\frac{16-2\sqrt{15}}{4}}=\frac{1}{\sqrt{15}-1}.\frac{\sqrt{15}-1}{2}=\frac{1}{2}$

Bài 12: Thực hiện các phép tính sau:

a) $\frac{\sqrt{15}-\sqrt{5}}{\sqrt{3}-1}-\frac{5-2\sqrt{5}}{2\sqrt{5}-4}$

b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}+\frac{\sqrt{3}-1}{\sqrt{3}+1}$

c) $\frac{2\sqrt{8}-\sqrt{12}}{\sqrt{18}-\sqrt{48}}-\frac{\sqrt{5}+\sqrt{27}}{\sqrt{30}-\sqrt{2}}$

d) $\frac{3-\sqrt{3}}{2\sqrt{3}-1}+\frac{3+\sqrt{3}}{2\sqrt{3}-1}$

e) $\frac{2}{\sqrt{3}-1}-\frac{2}{\sqrt{3}+1}$

f) $\frac{\sqrt{3}}{\sqrt{\sqrt{3}+1}-1}-\frac{\sqrt{3}}{\sqrt{\sqrt{3}+1}+1}$

g) $\frac{2\sqrt{3}-4}{\sqrt{3}-1}+\frac{2\sqrt{2}-1}{\sqrt{2}-1}-\frac{1+\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

h) $\frac{5}{12(2\sqrt{5}+3\sqrt{2})}-\frac{5}{12(2\sqrt{5}-3\sqrt{2})}$

HD:

a) $\frac{\sqrt{15}-\sqrt{5}}{\sqrt{3}-1}-\frac{5-2\sqrt{5}}{2\sqrt{5}-4}=\frac{\sqrt{5}.\left(\sqrt{3}-1\right)}{\sqrt{3}-1}-\frac{\left(5-2\sqrt{5}\right).\left(2\sqrt{5}+4\right)}{20-16}=\sqrt{5}-\frac{\sqrt{5}}{2}=\frac{\sqrt{5}}{2}$

b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}+\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{\left(\sqrt{3}+1\right)^2+\left(\sqrt{3}-1\right)^2}{3-1}=\frac{4+2\sqrt{3}+4-2\sqrt{3}}{2}=4$

c) $\frac{2\sqrt{8}-\sqrt{12}}{\sqrt{18}-\sqrt{48}}-\frac{\sqrt{5}+\sqrt{27}}{\sqrt{30}-\sqrt{2}}=\frac{4\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-4\sqrt{3}}-\frac{\sqrt{5}+3\sqrt{3}}{\sqrt{30}-\sqrt{2}}$

$=\frac{\left(4\sqrt{2}-2\sqrt{3}\right).\left(3\sqrt{2}+4\sqrt{3}\right)}{18-48}-\frac{\left(\sqrt{5}+3\sqrt{3}\right).\left(\sqrt{30}+\sqrt{2}\right)}{30-2}=\frac{-26\sqrt{6}-15\sqrt{10}}{42}$

d) $\frac{3-\sqrt{3}}{2\sqrt{3}-1}+\frac{3+\sqrt{3}}{2\sqrt{3}-1}=\frac{6}{2\sqrt{3}-1}=\frac{6.\left(2\sqrt{3}+1\right)}{12-1}=\frac{12\sqrt{3}+6}{11}$

e) $\frac{2}{\sqrt{3}-1}-\frac{2}{\sqrt{3}+1}=\frac{2.\left(\sqrt{3}+1\right)}{3-1}-\frac{2.\left(\sqrt{3}-1\right)}{3-1}=2$

f) $\frac{\sqrt{3}}{\sqrt{\sqrt{3}+1}-1}-\frac{\sqrt{3}}{\sqrt{\sqrt{3}+1}+1}=\frac{\sqrt{3}.\left(\sqrt{\sqrt{3}+1}+1\right)}{\sqrt{3}}-\frac{\sqrt{3}.\left(\sqrt{\sqrt{3}+1}-1\right)}{\sqrt{3}}=2$

g)

$\frac{2\sqrt{3}-4}{\sqrt{3}-1}+\frac{2\sqrt{2}-1}{\sqrt{2}-1}-\frac{1+\sqrt{6}}{\sqrt{2}+\sqrt{3}}=\frac{\left(2\sqrt{3}-4\right).\left(\sqrt{3}+1\right)}{3-1}+\frac{\left(2\sqrt{2}-1\right).\left(\sqrt{2}+1\right)}{2-1}-\frac{\left(1+\sqrt{6}\right).\left(\sqrt{2}-\sqrt{3}\right)}{2-3}$

$=\frac{6+2\sqrt{3}-4\sqrt{3}-4}{2}+\frac{4+2\sqrt{2}-\sqrt{2}-1}{1}-\frac{\sqrt{3}-\sqrt{2}+3\sqrt{2}-2\sqrt{3}}{1}=4-\sqrt{2}$

h) $\frac{5}{12.\left(2\sqrt{5}+3\sqrt{2}\right)}-\frac{5}{12.\left(2\sqrt{5}-3\sqrt{2}\right)}=\frac{5}{12}.\frac{2\sqrt{5}-3\sqrt{2}-2\sqrt{5}-3\sqrt{2}}{20-18}=\frac{-5\sqrt{2}}{4}$

Bài 13: Thực hiện các phép tính sau:

a) $\frac{1}{\sqrt{11-4\sqrt{7}}}-\frac{6}{\sqrt{32-10\sqrt{7}}}$

b) $\frac{1}{\sqrt{12-\sqrt{140}}}-\frac{1}{\sqrt{8-\sqrt{60}}}-\frac{1}{\sqrt{10+\sqrt{84}}}$

c) $\frac{1}{\sqrt{3}-\sqrt{2}}-\frac{2}{\sqrt{7}+\sqrt{5}}-\frac{3}{\sqrt{7-2\sqrt{10}}}+\frac{4}{\sqrt{10+2\sqrt{21}}}$

HD:

a) $\frac{1}{\sqrt{11-4\sqrt{7}}}-\frac{6}{\sqrt{32-10\sqrt{7}}}=\frac{1}{\sqrt{7}-2}-\frac{6}{5-\sqrt{7}}=\frac{\sqrt{7}+2}{7-4}-\frac{6.\left(5+\sqrt{7}\right)}{25-7}=\frac{2\sqrt{7}-23}{33}$

b) $\frac{1}{\sqrt{12-\sqrt{140}}}-\frac{1}{\sqrt{8-\sqrt{60}}}-\frac{1}{\sqrt{10+\sqrt{84}}}=\frac{1}{\sqrt{12-2\sqrt{35}}}-\frac{1}{\sqrt{8-2\sqrt{15}}}-\frac{1}{\sqrt{10+2\sqrt{21}}}$

$=\frac{1}{\sqrt{7}-\sqrt{5}}-\frac{1}{\sqrt{5}-\sqrt{3}}-\frac{1}{\sqrt{7}+\sqrt{3}}=\frac{\sqrt{7}+\sqrt{5}}{7-5}-\frac{\sqrt{5}+\sqrt{3}}{5-3}-\frac{\sqrt{7}-\sqrt{3}}{7-3}=\frac{\sqrt{7}-\sqrt{3}}{4}$

c) $\frac{1}{\sqrt{3}-\sqrt{2}}-\frac{2}{\sqrt{7}+\sqrt{5}}-\frac{3}{\sqrt{7-2\sqrt{10}}}+\frac{4}{\sqrt{10+2\sqrt{21}}}$

$=\frac{\sqrt{3}+\sqrt{2}}{3-2}-\frac{2.\left(\sqrt{7}-\sqrt{5}\right)}{7-5}-\frac{3}{\sqrt{5}-\sqrt{2}}+\frac{4}{\sqrt{7}+\sqrt{3}}$

$=\sqrt{3}+\sqrt{2}-\sqrt{7}+\sqrt{5}-\frac{3.\left(\sqrt{5}+\sqrt{2}\right)}{5-2}+\frac{4.\left(\sqrt{7}-\sqrt{3}\right)}{7-3}$

$=\sqrt{3}+\sqrt{2}-\sqrt{7}+\sqrt{5}-\sqrt{5}-\sqrt{2}+\sqrt{7}-\sqrt{3}=0$

Bài 14: Chứng minh các số sau đây là số nguyên:

a) $A=\frac{3\sqrt{3}+2\sqrt{2}}{\sqrt{3}+\sqrt{2}}+\frac{\sqrt{6}+6}{\sqrt{6}+1}$

b) $B=\left(\frac{15}{\sqrt{6}+1}+\frac{4}{\sqrt{6}-2}-\frac{12}{3-\sqrt{6}}\right)\left(\sqrt{6}+11\right)$

c) $C=\frac{2\sqrt{3+2\sqrt{3}-\sqrt{2}+\sqrt{3+2\sqrt{2}}}}{\sqrt{3}-1}-2\sqrt{3}$

HD:

a) $A=\dfrac{3\sqrt{3}+2\sqrt{2}}{\sqrt{3}+\sqrt{2}}+\dfrac{\sqrt{6}+6}{\sqrt{6}+1}=\dfrac{(3\sqrt{3}+2\sqrt{2})\left(\sqrt{3}-\sqrt{2}\right)}{(\sqrt{3}+\sqrt{2})\left(\sqrt{3}-\sqrt{2}\right)}+\dfrac{\sqrt{6}(\sqrt{6}+1)}{\sqrt{6}+1}=5$ (đpcm)

b) $B=\left(\dfrac{15}{\sqrt{6}+1}+\dfrac{4}{\sqrt{6}-2}-\dfrac{12}{3-\sqrt{6}}\right)\left(\sqrt{6}+11\right)$

$=\left(\dfrac{15\left(\sqrt{6}-1\right)}{\left(\sqrt{6}+1\right)\left(\sqrt{6}-1\right)}+\dfrac{4\left(\sqrt{6}+2\right)}{\left(\sqrt{6}-2\right)\left(\sqrt{6}+2\right)}-\dfrac{12\left(3+\sqrt{6}\right)}{\left(3-\sqrt{6}\right)\left(3+\sqrt{6}\right)}\right)\left(\sqrt{6}+11\right)$

$=\left(\dfrac{15\sqrt{6}-15}{5}+\dfrac{4\sqrt{6}+8}{2}-\dfrac{36+12\sqrt{6}}{3}\right)\left(\sqrt{6}+11\right)=\left(\sqrt{6}-11\right)\left(\sqrt{6}+11\right)=-115$ (đpcm)

c) $C=\dfrac{2\sqrt{3+2\sqrt{3}-\sqrt{2}+\sqrt{3+2\sqrt{2}}}}{\sqrt{3}-1}-2\sqrt{3}=\dfrac{2\sqrt{3+2\sqrt{3}-\sqrt{2}+\sqrt{\left(\sqrt{2}+1\right)^2}}}{\sqrt{3}-1}-2\sqrt{3}$

$=\dfrac{2\sqrt{3+2\sqrt{3}-\sqrt{2}+\sqrt{2}+1}}{\sqrt{3}-1}-2\sqrt{3}=\dfrac{2\sqrt{4+2\sqrt{3}}}{\sqrt{3}-1}-2\sqrt{3}$

$=\dfrac{2\left(\sqrt{3}+1\right)}{\sqrt{3}-1}-2\sqrt{3}=\dfrac{2\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}-2\sqrt{3}=4$ (đpcm)

Bài 15: Chứng minh các số sau đây là số dương:

a) $A=\dfrac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\dfrac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}$

b) $B=\dfrac{23\sqrt{2}}{\sqrt{2}+\sqrt{14+5\sqrt{3}}}+\dfrac{3\sqrt{2}}{\sqrt{2}-\sqrt{14-5\sqrt{3}}}$

c) $C=\dfrac{\sqrt{1+\dfrac{2\sqrt{2}}{3}}+\sqrt{1-\dfrac{2\sqrt{2}}{3}}}{\sqrt{1+\dfrac{2\sqrt{2}}{3}}-\sqrt{1-\dfrac{2\sqrt{2}}{3}}}$

HD:

a) $A=\dfrac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\dfrac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}$

$=\dfrac{\left(2+\sqrt{3}\right)\left(\sqrt{2}-\sqrt{2+\sqrt{3}}\right)}{\left(\sqrt{2}+\sqrt{2+\sqrt{3}}\right)\left(\sqrt{2}-\sqrt{2+\sqrt{3}}\right)}+\dfrac{\left(2-\sqrt{3}\right)\left(\sqrt{2}+\sqrt{2-\sqrt{3}}\right)}{\left(\sqrt{2}-\sqrt{2-\sqrt{3}}\right)\left(\sqrt{2}+\sqrt{2-\sqrt{3}}\right)}$

$$= \frac{-2\sqrt{2}+2\sqrt{2+\sqrt{3}}-\sqrt{6}+\sqrt{3}\sqrt{2+\sqrt{3}}}{\sqrt{3}} + \frac{2\sqrt{2}+2\sqrt{2+\sqrt{3}}-\sqrt{6}-\sqrt{3}\sqrt{2+\sqrt{3}}}{\sqrt{3}}$$

$$= \frac{4\sqrt{2+\sqrt{3}}-2\sqrt{6}}{\sqrt{3}} > 0$$

b) $B = \frac{23\sqrt{2}}{\sqrt{2}+\sqrt{14+5\sqrt{3}}} + \frac{3\sqrt{2}}{\sqrt{2}-\sqrt{14-5\sqrt{3}}}$

$$= \frac{46}{2+\sqrt{28+10\sqrt{3}}} + \frac{6}{2-\sqrt{28-10\sqrt{3}}}$$

$$= \frac{46}{2+5+\sqrt{3}} + \frac{6}{2-5+\sqrt{3}} = \frac{46}{7+\sqrt{3}} + \frac{6}{\sqrt{3}-3}$$

$$= \frac{46\left(7-\sqrt{3}\right)}{49-3} + \frac{6\left(\sqrt{3}+3\right)}{3-9} = 7-\sqrt{3}-\sqrt{3}-3 = 4-2\sqrt{3} > 0$$

c) $C = \frac{\sqrt{1+\frac{2\sqrt{2}}{3}}+\sqrt{1-\frac{2\sqrt{2}}{3}}}{\sqrt{1+\frac{2\sqrt{2}}{3}}-\sqrt{1-\frac{2\sqrt{2}}{3}}}$

$$= \frac{\left(\sqrt{1+\frac{2\sqrt{2}}{3}}+\sqrt{1-\frac{2\sqrt{2}}{3}}\right)^2}{\left(\sqrt{1+\frac{2\sqrt{2}}{3}}+\sqrt{1-\frac{2\sqrt{2}}{3}}\right)\left(\sqrt{1+\frac{2\sqrt{2}}{3}}-\sqrt{1-\frac{2\sqrt{2}}{3}}\right)}$$

$$= \frac{6+2\sqrt{5}}{3}.\frac{3}{4\sqrt{2}} = \frac{3+\sqrt{5}}{2} > 0$$

Bài 16: Chứng tỏ rằng các số sau là số hữu tỉ:

a) $\frac{2}{\sqrt{7}-5} - \frac{2}{\sqrt{7}+5}$ b) $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} + \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$

HD:

a) $\frac{2}{\sqrt{7}-5} - \frac{2}{\sqrt{7}+5} = \frac{2\left(\sqrt{7}+5\right)}{\left(\sqrt{7}-5\right)\left(\sqrt{7}+5\right)} - \frac{2(\sqrt{7}-5)}{(\sqrt{7}+5)\left(\sqrt{7}-5\right)} = \frac{-10}{9}$ (đpcm)

b) $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} + \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{\left(\sqrt{7}+\sqrt{5}\right)^2}{\left(\sqrt{7}-\sqrt{5}\right)\left(\sqrt{7}+\sqrt{5}\right)} + \frac{\left(\sqrt{7}-\sqrt{5}\right)^2}{\left(\sqrt{7}-\sqrt{5}\right)\left(\sqrt{7}+\sqrt{5}\right)} = \frac{27}{11}$ (đpcm)

Bài 17:Chứng minh rằng

a. $\frac{a\sqrt{a}+b\sqrt{b}}{\sqrt{a}+\sqrt{b}}-\sqrt{ab}=(\sqrt{a}-\sqrt{b})^2(a,b>0)$ b. $\frac{\sqrt{a}}{\sqrt{a}-\sqrt{b}}-\frac{\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\frac{a+b}{a-b}(a,b\geq 0;a\neq b)$

c. $\frac{(a\sqrt{b}+b)(\sqrt{a}+\sqrt{b})}{a-b}.\sqrt{\frac{ab+b^2-2\sqrt{ab^3}}{a(a+2\sqrt{b})+b}}=b(a,b>0)$

HD:

a. $\frac{a\sqrt{a}+b\sqrt{b}}{\sqrt{a}+\sqrt{b}}-\sqrt{ab}=\frac{(\sqrt{a})^3+(\sqrt{b})^3}{\sqrt{a}+\sqrt{b}}-\sqrt{ab}=a-\sqrt{ab}+b-\sqrt{ab}=(\sqrt{a}-\sqrt{b})^2$

b. $\frac{\sqrt{a}}{\sqrt{a}-\sqrt{b}}-\frac{\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\frac{\sqrt{a}(\sqrt{a}+\sqrt{b})-\sqrt{b}(\sqrt{a}-\sqrt{b})}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}=\frac{a+b}{a-b}$

$\frac{(a\sqrt{b}+b)(\sqrt{a}+\sqrt{b})}{a-b}=\frac{\sqrt{b}(a+\sqrt{b})(\sqrt{a}+\sqrt{b})}{a-b};$

c. $\sqrt{\frac{b[(\sqrt{a})^2-2\sqrt{ab}+(\sqrt{b})^2]}{a^2+2a\sqrt{b}+(\sqrt{b})^2}}=\sqrt{\frac{b(\sqrt{a}-\sqrt{b})^2}{(a+\sqrt{b})^2}}=\frac{\sqrt{b}(\sqrt{a}-\sqrt{b})}{a+\sqrt{b}}$

$\Rightarrow C=b$

Dạng 6: Sử dụng các phép biến đổi căn thức bậc hai để giải phương trình

Phương pháp giải:

+) Đặt điều kiện để phương trình có nghĩa: $\sqrt{A}$ có nghĩa khi $A \geq 0$

+) Đưa thừa số ra ngoài dấu căn: $\sqrt{A^2B}=|A|\sqrt{B}(B\geq 0)=\begin{cases}A\sqrt{B}(A\geq 0;B\geq 0)\\-A\sqrt{B}(A<0;B\geq 0)\end{cases}$

+) Rút gọn các căn thức đồng dạng

+) Biến đổi phương trình về dạng: $\sqrt{A}=B\Leftrightarrow A=B^2(B\geq 0)$

Bài 1: Giải các phương trình sau:

a) $2\sqrt{3x}-4\sqrt{3x}=27-3\sqrt{3x}$ b) $3\sqrt{2x}-5\sqrt{8x}+7\sqrt{18x}=28$

c) $\sqrt{x^2-9}-3\sqrt{x-3}=0$ d) $\sqrt{x^2-4}-2\sqrt{x+2}=0$

HD:

a) $2\sqrt{3x}-4\sqrt{3x}=27-3\sqrt{3x}$

Điều kiện: $x\ge 0$

$2\sqrt{3x}-4\sqrt{3x}=27-3\sqrt{3x}\Leftrightarrow 2\sqrt{3x}-4\sqrt{3x}+3\sqrt{3x}=27$

$\Leftrightarrow \sqrt{3x}=27\Leftrightarrow 3x=729\Leftrightarrow x=243(TM)$

Vậy $S=\{243\}$

b) Điều kiện: $x\ge 0$

$3\sqrt{2x}-5\sqrt{8x}+7\sqrt{18x}=28\Leftrightarrow 3\sqrt{2x}-10\sqrt{2x}+21\sqrt{2x}=28$

$\Leftrightarrow 14\sqrt{2x}=28\Leftrightarrow \sqrt{2x}=2\Leftrightarrow 2x=4\Leftrightarrow x=2(TM)$

Vậy $S=\{2\}$

c) $\sqrt{x^2-9}-3\sqrt{x-3}=0$

ĐKXĐ: $\begin{cases}x^2-9\ge 0\\ x-3\ge 0\end{cases}\Leftrightarrow\begin{cases}\left[\begin{array}{l}x\ge 3\\ x\le -3\end{array}\right.\\ x\ge 3\end{cases}\Rightarrow x\ge 3$

$\sqrt{x^2-9}-3\sqrt{x-3}=0\Leftrightarrow\sqrt{(x-3)(x+3)}-3\sqrt{x-3}=0$

$\Leftrightarrow\sqrt{x-3}\left(\sqrt{x+3}-3\right)=0\Leftrightarrow\left[\begin{array}{l}\sqrt{x-3}=0\\ \sqrt{x+3}-3=0\end{array}\right.$

$\Leftrightarrow\left[\begin{array}{l}x=3\\ x+3=9\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=3(TM)\\ x=6(TM)\end{array}\right.$

$S=\{3;6\}$

d) $\sqrt{x^2-4}-2\sqrt{x+2}=0$

ĐKXĐ: $\begin{cases}x^2-4\ge 0\\ x+2\ge 0\end{cases}\Leftrightarrow\begin{cases}\left[\begin{array}{l}x\ge 2\\ x\le -2\end{array}\right.\\ x\ge -2\end{cases}\Rightarrow\left[\begin{array}{l}x\ge 2\\ x\le -2\end{array}\right.$

$\sqrt{x^2-4}-2\sqrt{x+2}=0\Leftrightarrow\sqrt{(x-2)(x+2)}-2\sqrt{x+2}=0$

$\Leftrightarrow\sqrt{x+2}\left(\sqrt{x-2}-2\right)=0\Leftrightarrow\left[\begin{array}{l}x=-2\\x-2=4\end{array}\right.\Leftrightarrow\left[\begin{array}{l}x=-2(TM)\\x=6(TM)\end{array}\right.$

$S=\{-2;6\}$

Bài 2: Giải phương trình:

a) $\sqrt{2x+3}=1+\sqrt{2}$ b) $\sqrt{x+1}=\sqrt{5}+3$ c) $\sqrt{3x-2}=2-\sqrt{3}$

HD:

$\sqrt{2x+3}=1+\sqrt{2}\left(DK:x\geq\frac{-3}{2}\right)$

a) $\Leftrightarrow 2x+3=1+2\sqrt{2}+2\Leftrightarrow 2x=2\sqrt{2}\Leftrightarrow x=\sqrt{2}(TM)$

$S=\left\{\sqrt{2}\right\}$

$\sqrt{x+1}=\sqrt{5}+3(DK:x\geq-1)$

b) $\Leftrightarrow x+1=5+6\sqrt{5}+9\Leftrightarrow x=13+6\sqrt{5}(TM)$

$S=\left\{13+6\sqrt{5}\right\}$

$\sqrt{3x-2}=2-\sqrt{3}\left(DK:x\geq\frac{2}{3}\right)$

$\Leftrightarrow 3x-2=4-4\sqrt{3}+3\Leftrightarrow 3x=9-4\sqrt{3}$

c) $\Leftrightarrow x=\frac{9-4\sqrt{3}}{3}$

$S=\left\{\frac{9-4\sqrt{3}}{3}\right\}$

Bài 3:Giải phương trình: $25\sqrt{\frac{a-3}{25}}-7\sqrt{\frac{4a-12}{9}}-7\sqrt{a^2-9}+18\sqrt{\frac{9a^2-81}{81}}=0$

HD:

$25\sqrt{\frac{a-3}{25}}-7\sqrt{\frac{4a-12}{9}}-7\sqrt{a^2-9}+18\sqrt{\frac{9a^2-81}{81}}=0\Leftrightarrow\frac{1}{3}\sqrt{a-3}-\sqrt{a^2-9}=0$

Cách 1: $\frac{1}{3}\sqrt{a-3}-\sqrt{a^2-9}=0\Leftrightarrow\sqrt{a-3}=3\sqrt{a^2-9}\Leftrightarrow\begin{cases}a\geq 3\\a-3=9\left(a^2-9\right)\end{cases}\Leftrightarrow a=3$

Cách 2: Điều kiện $a\geq 3$

Ta có: $\frac{1}{3}\sqrt{a-3}-\sqrt{a^2-9}=0\Leftrightarrow\sqrt{a-3}\left(\frac{1}{3}-\sqrt{a+3}\right)=0\Leftrightarrow\left[\begin{array}{l}a=3(tm)\\a=\frac{-26}{9}(loai)\end{array}\right.$

Bài 4:Giải phương trình: $\sqrt{18x+9}-\sqrt{8x+4}+\frac{1}{3}\sqrt{2x+1}=4$

HD: $\sqrt{18x+9}-\sqrt{8x+4}+\frac{1}{3}\sqrt{2x+1}=4\leftrightarrow\frac{2}{3}\sqrt{2x+1}=4\Leftrightarrow x=\frac{35}{2}$

Bài 5: Giải các phương trình sau

a. $\frac{3}{2}\sqrt{4x-8}-9\sqrt{\frac{x-2}{81}}=6$

b. $\sqrt{9x-9}-\sqrt{4x-4}+\sqrt{16x-16}-3\sqrt{x-1}=16(x\geq1)$

c. $\frac{1}{\sqrt{x^2+1}+1}-\frac{1}{\sqrt{x^2+1}-1}+2=0(x\geq-1;x\neq0)$

d. $\sqrt{36x-72}-15\sqrt{\frac{x-2}{25}}=4(5+\sqrt{x-2})(x\geq2)$

e. $\frac{1}{\sqrt{x+3}+\sqrt{x+2}}+\frac{1}{\sqrt{x+2}+\sqrt{x+1}}+\frac{1}{\sqrt{x+1}+\sqrt{x}}=1(x\geq0)$

HD:

a. $\frac{3}{2}\sqrt{4x-8}-9\sqrt{\frac{x-2}{81}}=6\Leftrightarrow\frac{3}{2}\sqrt{4(x-2)}-9\sqrt{\frac{x-2}{9^2}}=6\Leftrightarrow\frac{3}{2}.2.\sqrt{x-2}=9.\frac{1}{9}.\sqrt{x-2}=6$
$\Leftrightarrow\sqrt{x-2}=3\Leftrightarrow x=11$

b. $\sqrt{9x-9}-\sqrt{4x-4}+\sqrt{16x-16}-3\sqrt{x-1}=16\Leftrightarrow2\sqrt{x-1}=16\Leftrightarrow\sqrt{x-1}=8\Leftrightarrow x=65(tm)$

c. $\frac{1}{\sqrt{x^2+1}+1}-\frac{1}{\sqrt{x^2+1}-1}+2=0\Rightarrow(\sqrt{x^2+1}-1)-(\sqrt{x^2+1}+1)+2(\sqrt{x+1}-1)(\sqrt{x^2+1}+1)=0$
$\Leftrightarrow\sqrt{x^2+1}-1-\sqrt{x^2+1}-1+2(x+1-1)=0\Leftrightarrow-2+2x=0\Leftrightarrow x=1(tm)$

d. $\sqrt{36x-72}-15\sqrt{\frac{x-2}{25}}=4(5+\sqrt{x-2})\Leftrightarrow6\sqrt{x-2}-3\sqrt{x-2}=4(5+\sqrt{x-2})$
$\Leftrightarrow\sqrt{x-2}=-20\Rightarrow ptvn$

e. $\frac{1}{\sqrt{x+3}+\sqrt{x+2}}+\frac{1}{\sqrt{x+2}+\sqrt{x+1}}+\frac{1}{\sqrt{x+1}+\sqrt{x}}=1$
$\Leftrightarrow\sqrt{x+3}-\sqrt{x+2}+\sqrt{x+2}-\sqrt{x+1}+\sqrt{x+1}-\sqrt{x}=1$

$\Leftrightarrow x+3=x+2\sqrt{x}+1\Leftrightarrow \sqrt{x}=1\Leftrightarrow x=1(tm)$

C. BÀI TẬP VỀ NHÀ

Bài 1:Đưa thừa số ra ngoài dấu căn:

a) $\sqrt{5a^2}\,(a\le 0)$ b) $\sqrt{18a^2}\,(a\ge 0)$ c) $\sqrt{-9b^3}\,(b\le 0)$ d) $\sqrt{24a^4b^8}\,(a,b\in R)$

HD:

a) $\sqrt{5a^2}\,(a\le 0)=5|a|=-a\sqrt{5}\,(a\le 0)$ b) $\sqrt{18a^2}\,(a\ge 0)=18|a|=3a\sqrt{2}\,(a\ge 0)$

c) $\sqrt{-9b^3}\,(b\le 0)=-3b\sqrt{-b}$ d) $\sqrt{24a^4b^8}\,(a,b\in R)=2\sqrt{6}a^2b^4\,(a,b\in R)$

Bài 2:Đưa thừa số vào trong dấu căn:

a) $x\sqrt{7}\,(x\ge 0)$ b) $x\sqrt{15}\,(x\le 0)$ c) $\frac{1}{y}\sqrt{19y}\,(y>0)$ d) $\frac{1}{3}y\sqrt{\frac{27}{y^2}}\,(y\le 0)$

HD:

a) $x\sqrt{7}\,(x\ge 0)=\sqrt{7x^2}\,(x\ge 0)$ b) $x\sqrt{15}\,(x\le 0)=-\sqrt{15x^2}$

c) $\frac{1}{y}\sqrt{19y}\,(y>0)=\sqrt{\frac{19}{y}}\,(y>0)$

d) $\frac{1}{3}y\sqrt{\frac{27}{y^2}}\,(y\le 0)=-\sqrt{3}\,(y\le 0)$

Bài 3:Tìm số lớn hơn trong các cặp số dưới đây

a) $2\sqrt{6}$ và $3\sqrt{3}$ b) $\frac{2}{5}\sqrt{6}$ và $\frac{7}{4}\sqrt{\frac{1}{3}}$

HD:

a) $\begin{cases}2\sqrt{6}=\sqrt{24}\\3\sqrt{3}=\sqrt{27}\end{cases}\Rightarrow 2\sqrt{6}<3\sqrt{3}$

b) $\frac{2}{5}\sqrt{6}=\sqrt{\frac{4}{25}.6}=\sqrt{\frac{24}{25}};\frac{7}{4}\sqrt{\frac{1}{3}}=\sqrt{\frac{49}{16}.\frac{1}{3}}=\sqrt{\frac{49}{48}}\Rightarrow\frac{2}{5}\sqrt{6}<\frac{7}{4}\sqrt{\frac{1}{3}}\left(\frac{24}{25}<1<\frac{49}{48}\right)$

Bài 4:Tìm số bé hơn trong các cặp số dưới đây

a) $2\sqrt{23}$ và $3\sqrt{10}$

b) $2\sqrt{\frac{1}{5}}$ và $\frac{1}{5}\sqrt{21}$

HD:

a) $\begin{cases}2\sqrt{23}=\sqrt{4.23}=\sqrt{92}\\3\sqrt{10}=\sqrt{3^2.10}=\sqrt{90}\end{cases}\Rightarrow 2\sqrt{23}>3\sqrt{10}$

b) $2\sqrt{\frac{1}{5}}=\sqrt{\frac{4}{5}};\frac{1}{5}\sqrt{21}=\frac{21}{25};\frac{4}{5}>\frac{21}{25}\Rightarrow 2\sqrt{\frac{1}{5}}>\frac{1}{5}\sqrt{21}$

Bài 5:Sắp xếp các số

a) $2\sqrt{5};3\sqrt{2};5;\sqrt{23}$ theo thứ tự tăng dần

b) $5\sqrt{2};2\sqrt{13};4\sqrt{3};\sqrt{47}$ theo thứ tự giảm dần

HD:

a) $3\sqrt{2}<2\sqrt{5}<\sqrt{23}<5$

b) $2\sqrt{13}>5\sqrt{2}>4\sqrt{3}>\sqrt{47}$

Bài 6: Rút gọn biểu thức

a. $A=4\sqrt{\frac{25x}{4}}-\frac{8}{3}\sqrt{\frac{9x}{4}}-\frac{4}{3x}\sqrt{\frac{9x^3}{64}}(x\ge 0)$

b. $B=\frac{y}{2}+\frac{3}{4}\sqrt{1-4y+4y^2}-\frac{3}{2}(y\le\frac{1}{2})$

HD:

a) $A=4\sqrt{\frac{25x}{4}}-\frac{8}{3}\sqrt{\frac{9x}{4}}-\frac{4}{3x}\sqrt{\frac{9x^3}{64}}(x\ge 0)=\frac{11}{2}\sqrt{x}$

b) $B=\frac{y}{2}+\frac{3}{4}\sqrt{1-4y+4y^2}-\frac{3}{2}(y\le\frac{1}{2})=-y-\frac{3}{4}$

Bài 7:Thực hiện phép tính

a. $A=(\frac{2}{\sqrt{3}-1}+\frac{3}{\sqrt{3}-2}+\frac{15}{3-\sqrt{3}}).\frac{1}{\sqrt{3}+5}$

b. $B=(\frac{\sqrt{14}-\sqrt{7}}{1-\sqrt{2}}+\frac{\sqrt{15}-\sqrt{5}}{1-\sqrt{3}}):\frac{1}{\sqrt{7}-\sqrt{5}}$

HD:

a) $A=(\frac{2}{\sqrt{3}-1}+\frac{3}{\sqrt{3}-2}+\frac{15}{3-\sqrt{3}}).\frac{1}{\sqrt{3}+5}=\frac{1}{2}$

b) $B=(\frac{\sqrt{14}-\sqrt{7}}{1-\sqrt{2}}+\frac{\sqrt{15}-\sqrt{5}}{1-\sqrt{3}}):\frac{1}{\sqrt{7}-\sqrt{5}}=-2$

Bài 8: Tìm u, biết

a. $\sqrt{4u-20}+3\sqrt{\frac{u-5}{9}}-\frac{1}{3}\sqrt{9u-45}=4$

b. $\frac{2}{3}\sqrt{9u-9}-\frac{1}{4}\sqrt{16u-16}+27\sqrt{\frac{u-1}{81}}=4$

HD:

a) $\sqrt{4u-20}+3\sqrt{\frac{u-5}{9}}-\frac{1}{3}\sqrt{9u-45}=4\Leftrightarrow 2\sqrt{u-5}=4\Leftrightarrow u=9$

b) $\frac{2}{3}\sqrt{9u-9}-\frac{1}{4}\sqrt{16u-16}+27\sqrt{\frac{u-1}{81}}=4\Leftrightarrow 4\sqrt{u-1}=4\Leftrightarrow u=2$

Bài 9:Tìm x, y, z biết: $\sqrt{x+1}+\sqrt{y-3}+\sqrt{z-1}=\frac{1}{2}(x+y+z)$

HD:

Cách 1: $\sqrt{x+1}+\sqrt{y-3}+\sqrt{z-1}=\frac{1}{2}(x+y+z)\Leftrightarrow\left(\sqrt{x+1}-1\right)^2+\left(\sqrt{y-3}-1\right)^2+\left(\sqrt{z-1}-1\right)^2=0\Rightarrow\begin{cases}x=0\\y=4\\z=2\end{cases}$

Cách 2: Ta có: $x+2=(x+1)+1\geq 2\sqrt{x+1};y-2=(y-3)+1\geq 2\sqrt{y-3};z=(z-1)+1\geq 2\sqrt{z-1}$

Cộng vế các bất đẳng thức ta được: $x+y+z\geq 2\left(\sqrt{x+1}+\sqrt{y-3}+\sqrt{z-1}\right)\Leftrightarrow x=0;y=4;z=2$

Bài 10: Chứng minh rằng: $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+...+\frac{1}{\sqrt{n-1}+\sqrt{n}}=\sqrt{n}-1$

HD:

Thực hiện trục căn thức ở mẫu với từng thừa số

$$\frac{1}{\sqrt{1}+\sqrt{2}}=\frac{\sqrt{1}-\sqrt{2}}{1-2}=\sqrt{2}-1;\frac{1}{\sqrt{2}+\sqrt{3}}=\sqrt{3}-\sqrt{2};.....$$

Thực hiện rút gọn ta được: VT = $\sqrt{n}-1=VP$